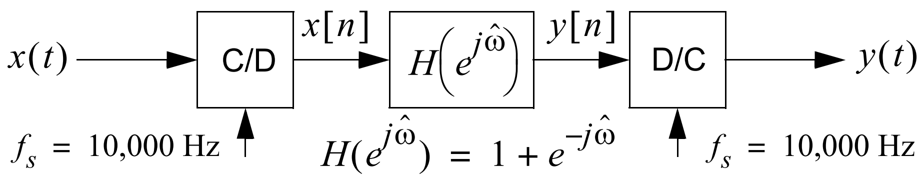
Filtering Sampled Continuous-Time Signals



Consider the above analog input/analog output DSP system with

$$x(t) = 7\cos(2\pi \cdot 500 \cdot t) + 20\cos(2\pi \cdot 2000 \cdot t).$$

- Find $x[n]$, $y[n]$, and $y(t)$

- We start by noting the frequencies of the input sinusoids
- We have frequencies at 500 Hz and 2000 Hz, both of which are less than 10,000/2 = 5,000 Hz, the folding frequency, thus 500 and 2000 are principle alias values
- The final output frequencies will also be 500 and 2000 Hz
- We now convert $f_1 = 500$ and $f_2 = 2000$ to the respective $\hat{\omega}$ frequencies

$$\hat{\omega}_1 = 2\pi \cdot \frac{500}{10000} = 0.1\pi \qquad \hat{\omega}_2 = 2\pi \cdot \frac{2000}{10000} = 0.4\pi$$

- We can now write $x[n]$ by making the substututiont $t \to nT_s = n/f_s$ or equivalently replacing $2\pi f_i t$ with $\hat{\omega}_i n$

$$x[n] = 7\cos(0.1\pi n) + 20\cos(0.4\pi n)$$

- Moving on, to obtain $y[n]$ we need to first obtain $H(e^{j\hat{\omega}_1})$ and $H(e^{j\hat{\omega}_2})$
- By direct evaluation (using a calculator or MATLAB or Mathematica)

$$H(e^{j\hat{\omega}_1}) = 1 - e^{-j0.1\pi} = 0.3129e^{j1.4137}$$

$$H(e^{j\hat{\omega}_2}) = 1 - e^{-j0.4\pi} = 1.1756e^{j0.9425}$$

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>> [abs(1-exp(-j*0.1*pi)) angle(1-exp(-j*0.1*pi))]
ans =    0.3129    1.4137

>> [abs(1-exp(-j*0.4*pi)) angle(1-exp(-j*0.4*pi))]
ans =   1.1756    0.9425
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- We now apply these magnitude and phase values to the sinusoids of $x[n]$, term-by-term, that is

$$x[n] = A\cos(\hat{\omega}_i n) \Rightarrow A\left|H(e^{j\hat{\omega}_i})\right|\cos(\hat{\omega}_i n + \angle H(e^{j\hat{\omega}_i})),\ i = 1, 2$$

- The result is

$$y[n] = 0.3129 \cdot 7\cos(0.1\pi n + 1.4137) + 1.1756 \cdot 20\cos(0.4\pi n + 0.9425)$$
$$= 2.1903\cos(0.1\pi n + 1.4137) + 23.512\cos(0.4\pi n + 0.9425)$$

- To convert back to the time domain we need to reverse the earlier steps and let $nT_s = n/f_s \to t$ or let $\hat{\omega}_i n$ be replaced by $2\pi f_i t$

$$y(t) = 2.1903\cos(2\pi \cdot 500 \cdot t + 1.4137) + 23.512\cos(2\pi \cdot 2000 \cdot t + 0.9425)$$