Fourier Series Example

Consider the signal

$$x(t) = 5 + 16\cos\left(2\pi \cdot 500 \cdot t - \frac{\pi}{4}\right) + 10\sin\left(2\pi \cdot 5000 \cdot t + \frac{\pi}{8}\right)$$

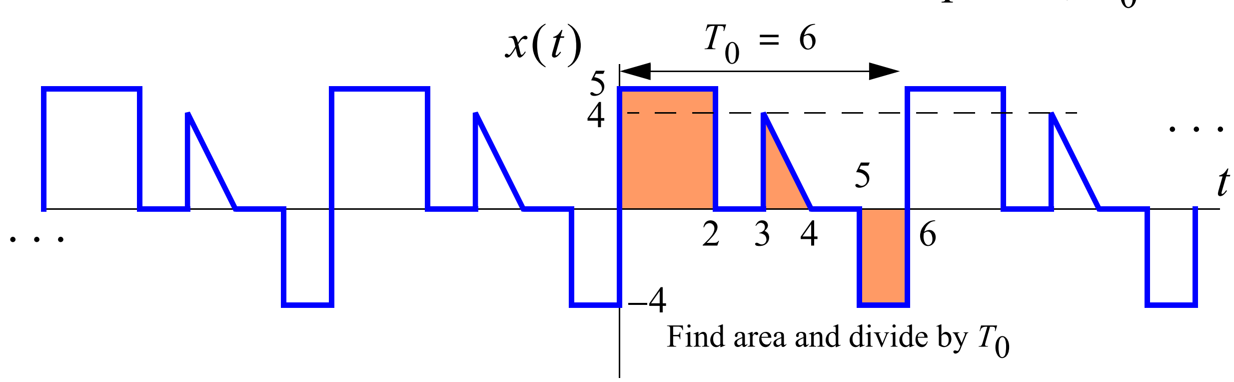
- Find the Fourier series coefficients, $\{a_k\}$; which harmonics are present?

- By inspection the fundamental frequency is GCD[500, 5000] = 500 Hz, so the harmonics present are 500/500 = 1 and 5000/500 = 10
- The Fourier coefficients we need to find are a_0, $\pm a_1$, and $\pm a_{10}$, **all others are zero**
- By inspection $a_0 = 5$
- To obtain the remaining $\{a_k\}$ we use inverse Eulers on each of the sinusoid terms
- By expanding we have

$$16\cos\left(2\pi(100)t - \frac{\pi}{4}\right) = \frac{16}{2}[e^{j(2\pi(500)t - \pi/4)} + e^{-j(2\pi(500)t - \pi/4)}] \Rightarrow a_{\pm 1} = 8e^{\mp j\pi/4}$$

$$10\sin\left(2\pi(5000)t + \frac{\pi}{8}\right) = \frac{10}{2}\left[e^{j\left(2\pi(5000)t - \frac{3\pi}{8}\right)} + e^{-j\left(2\pi(5000)t - \frac{3\pi}{8}\right)}\right] \Rightarrow a_{\pm 10} = 5e^{\mp j3\pi/4}$$

- For the periodic waveform shown below find the a_0 Fourier series coefficient and also find the waveform period, T_0



- The waveform period by inspection is $T_0 = 6\text{s}$
- From the definition

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt = \frac{1}{6}[5 \cdot 2 + (4 \cdot 1)/2 - (4 \cdot 1)] = \frac{10 + 2 - 4}{6} = \frac{4}{3}$$