Frequency Response Example

An LTI system has impulse response

$$h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2].$$

- Find the frequency response $H(e^{j\hat{\omega}})$
- In particular find $H(e^{j\hat{\omega}})$ when $\hat{\omega} = \pi/4$
- Is $H(e^{j\hat{\omega}}) = 0$ for any $\hat{\omega} \in [0, \pi]$?

- Given the impulse response, we know that the frequency response is just

$$H(e^{j\hat{\omega}}) = \sum_{n=0}^{M} h[n]e^{-j\hat{\omega}n}$$

- Here we have then

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}[e^{j\hat{\omega}} + 3 + e^{-j\hat{\omega}}] \\ &= e^{-j\hat{\omega}}[2\cos(\hat{\omega}) + 3] \end{aligned}$$

- At $\hat{\omega} = \pi/4$ we have

$$\begin{aligned} H(e^{j\pi/4}) &= 1 + 3e^{-j\pi/4} + e^{-j\pi/2} \\ &= \left[1 + 3\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{2}\right)\right] - j\left[3\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right)\right] \\ &= \left[1 + \frac{3}{\sqrt{2}}\right] - j\left[1 + \frac{3}{\sqrt{2}}\right] = 4.414e^{-j0.7854} \end{aligned}$$

- To consider if $H(e^{j\hat{\omega}})$ is zero anywhere on $[0, \pi]$ we can just look at the final form

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}[2\cos(\hat{\omega}) + 3]$$

- We see that the term $2\cos(\hat{\omega}) + 3$ can never be zero, so there are no $\hat{\omega}$ values in this case where the filter gain is zero