# LTI System Properties Example

Determine if the system

$$y[n] = x[n]\cos(0.2\pi n)$$

is (1) linear (2) time invariant

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- To check both linearity and time invariance we follow the proof templates in the text/notes

## Linearity:

- Form $w[n]$

$$w[n] = \alpha\{x_1[n]\cos(0.2\pi n)\} + \beta\{x_2[n]\cos(0.2\pi n)\}$$

- Form $y[n]$ with $x[n] = \alpha x_1[n] + \beta x_2[n]$

$$y[n] = \{\alpha x_1[n] + \beta x_2[n]\}\cos(0.2\pi n)$$
$$= \alpha x_1[n]\cos(0.2\pi n) + \beta x_2[n]\cos(0.2\pi n)$$

- The system **is linear** since $w[n] = y[n]$

## Time Invariance

- Form $w[n]$ (delayed input)

$$w[n] = \left\{ x[n]\big|_{n \to n - n_0} \right\}\cos(0.2\pi n) = x[n - n_0]\cos(0.2\pi n)$$

- Form $y[n - n_0]$

$$y[n - n_0] = \{x[n]\cos(0.2\pi n)\}\big|_{n \to n - n_0} = x[n - n_0]\cos(0.2\pi(n - n_0))$$

- We see that $w[n]$ does not equal $y[n - n_0]$, so the system is **not time invariant**

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Two system are connected in cascade, that is the output of $\mathcal{S}_1$ is connected into the input of $\mathcal{S}_2$

$$\mathcal{S}_1 : y_1[n] = x_1[n] + x_1[n-2] - x_1[n-3]$$
$$\mathcal{S}_2 : y_2[n] = x_2[n] + 2x_2[n-1]$$

- Find the impulse response, $h[n]$, of the cascade

## • Draw the *direct form* block diagram for the first system $\mathcal{S}_1$

- To find the impulse response of a two subsystem cascade, we need to convolve the individual impulse responses, i.e., form $h[n] = h_1[n] + h_2[n]$
- By inspection the impulse response of $\mathcal{S}_1$ is

$$h_1[n] = \delta[n] + \delta[n-2] - \delta[n-3] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 0, 1, -1\}$$

- By inspection the impulse response of $\mathcal{S}_2$ is

$$h_2[n] = \delta[n] + 2\delta[n-2] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 2\}$$

- We can perform the convolution using a table

sum of products formed between $h[k]$ and $x[n-k]$ inside red box.

Expected output range $[0+0, 3+1] = [0,4]$

| | | | | | | | | | | | | | | | | | $h[n]$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $h1[k]$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| $n = 0$ | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $n = 1$ | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| $n = 2$ | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $n = 3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $n = 4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -2 |
| $n = 4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

Flipped and shifted: $h2[n-k]$

Outputs for $n < 0$ and $n > 4$ are all 0

MATLAB Check

```
>> filter([1 0 1 -1],1,[1 2 0 0 0 0])
ans =     1    2    1    1   -2    0
         n=0                n=4
```

- The direct form block diagram of $\mathcal{S}_1$ is follows from the text/notes