

# LTI System Properties Example

Determine if the system

$$y[n] = x[n] \cos(0.2\pi n)$$

is (1) linear (2) time invariant

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- To check both linearity and time invariance we follow the proof templates in the text/notes

## Linearity:

- Form  $w[n]$

$$w[n] = \alpha \{x_1[n] \cos(0.2\pi n)\} + \beta \{x_2[n] \cos(0.2\pi n)\}$$

- Form  $y[n]$  with  $x[n] = \alpha x_1[n] + \beta x_2[n]$

$$\begin{aligned} y[n] &= \{\alpha x_1[n] + \beta x_2[n]\} \cos(0.2\pi n) \\ &= \alpha x_1[n] \cos(0.2\pi n) + \beta x_2[n] \cos(0.2\pi n) \end{aligned}$$

- The system is **linear** since  $w[n] = y[n]$

## Time Invariance

- Form  $w[n]$  (delayed input)

$$w[n] = \left\{ x[n] \Big|_{n \rightarrow n-n_0} \right\} \cos(0.2\pi n) = x[n-n_0] \cos(0.2\pi n)$$

- Form  $y[n-n_0]$

$$y[n-n_0] = \{x[n] \cos(0.2\pi n)\} \Big|_{n \rightarrow n-n_0} = x[n-n_0] \cos(0.2\pi(n-n_0))$$

- We see that  $w[n]$  does not equal  $y[n-n_0]$ , so the system is **not time invariant**

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Two system are connected in cascade, that is the output of  $S_1$  is connected into the input of  $S_2$

$$S_1: y_1[n] = x_1[n] + x_1[n-2] - x_1[n-3]$$

$$S_2: y_2[n] = x_2[n] + 2x_2[n-1]$$

- Find the impulse response,  $h[n]$ , of the cascade

- Draw the *direct form* block diagram for the first system  $\mathcal{S}_1$

- To find the impulse response of a two subsystem cascade, we need to convolve the individual impulse responses, i.e., form  $h[n] = h_1[n] + h_2[n]$
- By inspection the impulse response of  $\mathcal{S}_1$  is

$$h_1[n] = \delta[n] + \delta[n-2] - \delta[n-3] = \{1, 0, 1, -1\}$$

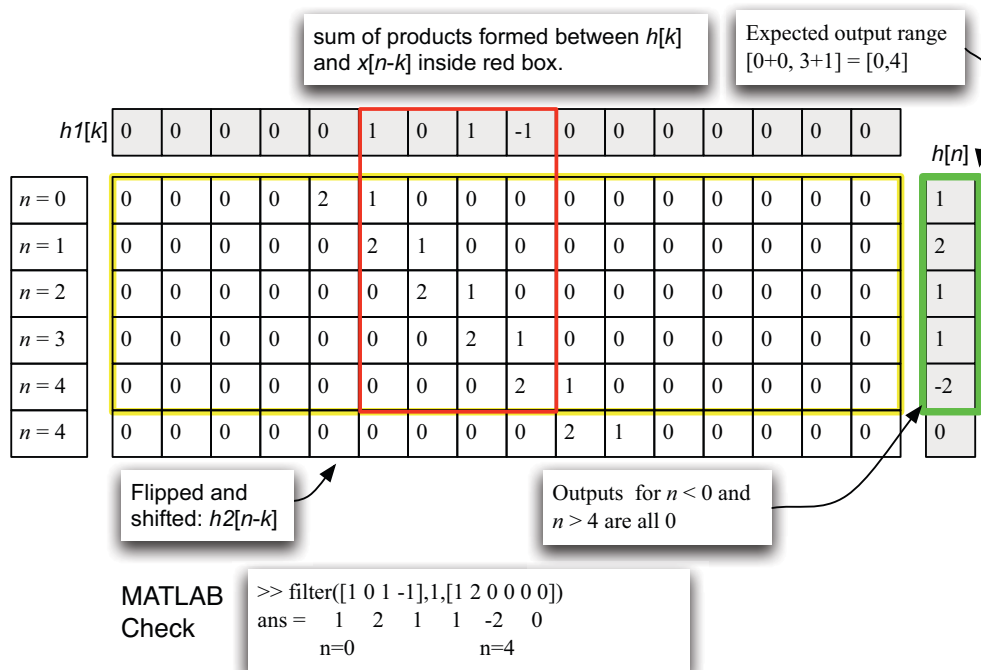
$n \uparrow$   
 $n = 0$

- By inspection the impulse response of  $\mathcal{S}_2$  is

$$h_2[n] = \delta[n] + 2\delta[n-2] = \{1, 2\}$$

$n \uparrow$   
 $n = 0$

- We can perform the convolution using a table



- The direct form block diagram of  $\mathcal{S}_1$  is follows from the text/notes

