Phasor Addition Example #1

- Consider the signal

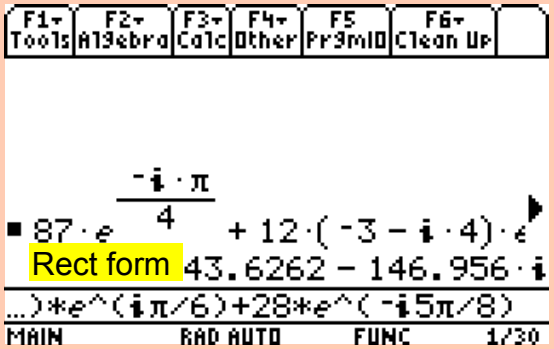
$$x(t) = 87\cos(2\pi \cdot 880 \cdot t - \pi/4) + 12\mathrm{Re}\{(-3 - j4)e^{j(2\pi \cdot 880 \cdot t + \pi/6)}\} + 28\sin(2\pi \cdot 880 \cdot t - \pi/8)$$

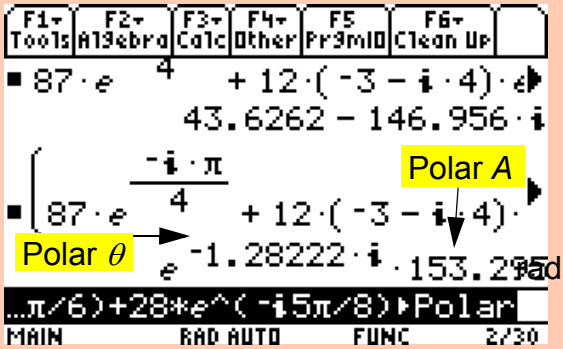
- Find $X = Ae^{j\theta}$ such that $x(t) = A\cos(2\pi \cdot 880 \cdot t + \theta)$

By inspection

$$X = 87e^{-j\pi/4} + 12(-3 - j4)e^{j(\pi/6)} + 28e^{-j(\pi/8 + \pi/2)}$$

- Note $\sin(\theta) = \cos(\theta - \pi/2)$
- To obtain a numerical solution for X we may simply enter values into a calculator





- Working out more of the lower level steps, we can start by writing

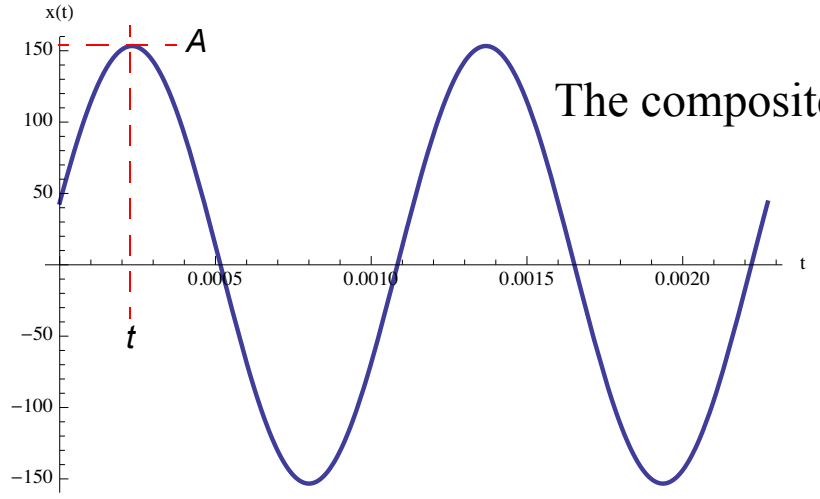
$$X = 87\{\cos(-\pi/4) + j\sin(-\pi/4)\} + 12\{-3 - j4\}\{\cos(\pi/6) + j\sin(\pi/6)\} + 28\{\cos(-5\pi/8) + j\sin(-5\pi/8)\}$$

- Evaluating the cos and sin terms we have

$$X = \{61.518 - j61.518\} + \{-7.177 - j59.569\} + \{-10.715 - j25.869\}$$
$$= \{-43.626 - j146.956\} = 153.295 e^{-j1.282}$$

- The direct calculation and the indirect calculation are in agreement
- Here a Mathematica plot (could also have been MATLAB) of the actual time domain waveform is used to *experimentally* determine A and ϕ

```
Plot[87 Cos[2 π 880 t - π / 4] + 12 Re[(-3 - j 4) e^(j (2 π 880 t+π/6))] + 28 Sin[2 π 880 t - π / 8],
 {t, 0, 2 / 880}, PlotStyle → Thick, AxesLabel → {"t", "x(t)"}]
```



The composite waveform

```
FindMaximum[87 Cos[2 π 880 t - π / 4] +
  12 Re[(-3 - j 4) e^(j (2 π 880 t+π/6))] + 28 Sin[2 π 880 t - π / 8], {t, 0.0002}]
{153.295, {t → 0.000231892}}
```

A

```
ϕ = -2 π (0.00023189169947837138` / (1 / 880))
-1.28218
```

ϕ

The results agree

Phasor Addition Example #2

- Consider the signal

$$x(t) = 30\cos(2\pi \cdot f_o \cdot t - \pi/2) + B\cos(2\pi \cdot f_o \cdot t + \phi) + 60\sin(2\pi \cdot f_o \cdot t + \pi/4)$$

- Find $X_B = Be^{j\phi}$ such that $x(t) = 50\cos(2\pi \cdot f_o \cdot t + \pi/4)$

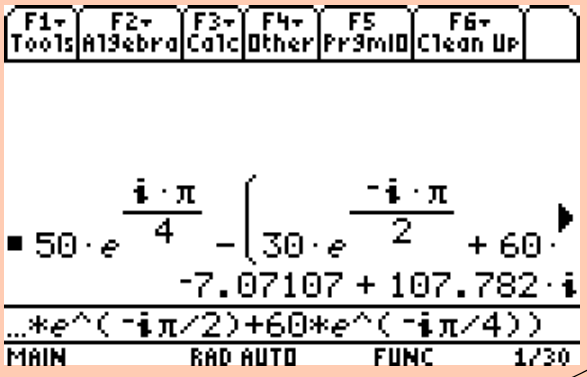
- We observe that this is just a avariation on the previous problem
- We start by writing

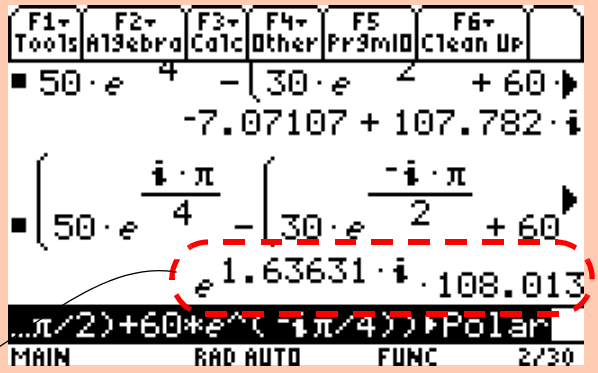
$$X = 50e^{j\pi/4} = 30e^{-j\pi/2} + Be^{j\phi} + 60e^{j(\pi/4 - \pi/2)}$$

so

$$X_B = Be^{j\phi} = 50e^{j\pi/4} - [30e^{-j\pi/2} + 60e^{-j\pi/4}]$$

- Via a TI-89





$B = 108.013$ $\phi = 1.636$ rad

- A lower level analysis can also be performed, as with #1

$$X_B = 50\left[\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right]$$

$$- 30\left[\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right] - 60\left[\cos\left(\frac{\pi}{4}\right) - j\sin\left(\frac{\pi}{4}\right)\right]$$

$$= [35.36 + j35.36] + [j30] + [-42.43 + j43.43]$$

$$= -7.07 + j107.78 = 108.013e^{j1.6363}$$

- The results agree