

Phasor Addition Example #1

- Consider the signal

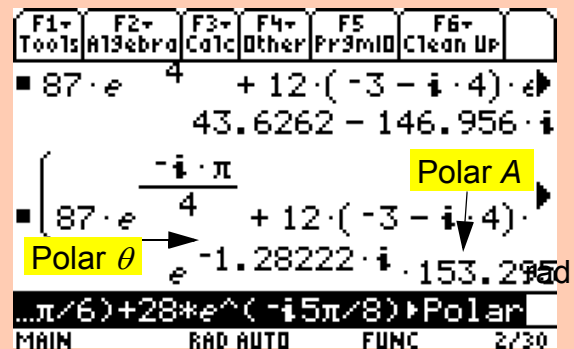
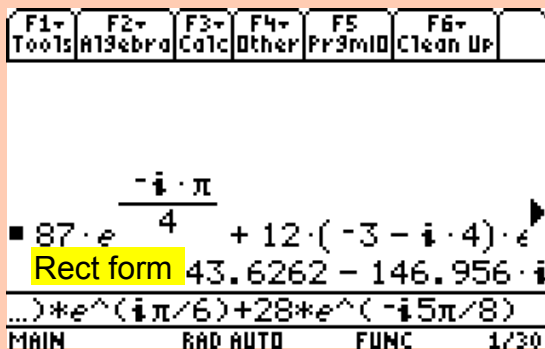
$$x(t) = 87 \cos(2\pi \cdot 880 \cdot t - \pi/4) + 12 \operatorname{Re}\{(-3 - j4)e^{j(2\pi \cdot 880 \cdot t + \pi/6)}\} + 28 \sin(2\pi \cdot 880 \cdot t - \pi/8)$$

- Find $X = Ae^{j\theta}$ such that $x(t) = A \cos(2\pi \cdot 880 \cdot t + \theta)$

By inspection

$$X = 87e^{-j\pi/4} + 12(-3 - j4)e^{j(\pi/6)} + 28e^{-j(\pi/8 + \pi/2)}$$

- Note $\sin(\theta) = \cos(\theta - \pi/2)$
- To obtain a numerical solution for X we may simply enter values into a calculator



- Working out more of the lower level steps, we can start by writing

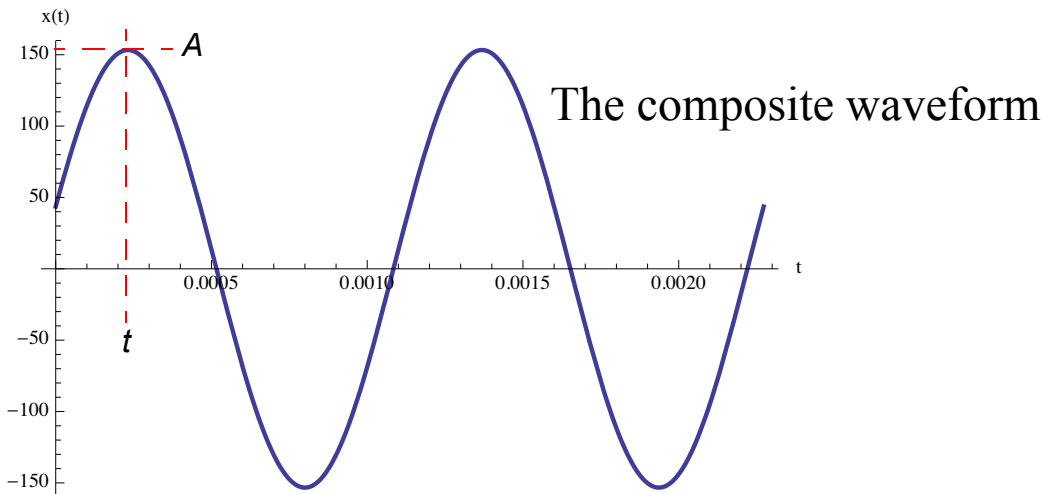
$$X = 87 \{ \cos(-\pi/4) + j \sin(-\pi/4) \} + 12 \{ -3 - j4 \} \{ \cos(\pi/6) + j \sin(\pi/6) \} + 28 \{ \cos(-5\pi/8) + j \sin(-5\pi/8) \}$$

- Evaluating the cos and sin terms we have

$$\begin{aligned}
 X &= \{61.518 - j61.518\} + \{-7.177 - j59.569\} \\
 &\quad + \{-10.715 - j25.869\} \\
 &= \{-43.626 - j146.956\} = 153.295 e^{-j1.282}
 \end{aligned}$$

- The direct calculation and the indirect calculation are in agreement
- Here a Mathematica plot (could also have been MATLAB) of the actual time domain waveform is used to *experimentally* determine A and ϕ

```
Plot[87 Cos[2 π 880 t - π / 4] + 12 Re[(-3 - j 4) e^{j (2 π 880 t + π / 6)}] + 28 Sin[2 π 880 t - π / 8],
{t, 0, 2 / 880}, PlotStyle -> Thick, AxesLabel -> {"t", "x(t)"}]
```



```
FindMaximum[87 Cos[2 π 880 t - π / 4] +
12 Re[(-3 - j 4) e^{j (2 π 880 t + π / 6)}] + 28 Sin[2 π 880 t - π / 8], {t, 0.0002}]
```

```
{153.295, {t -> 0.000231892}}
```

$$\phi = -2\pi \left(\frac{0.00023189169947837138}{1/880} \right)$$

```
-1.28218
```

A

The results agree

ϕ

Phasor Addition Example #2

- Consider the signal

$$x(t) = 30 \cos(2\pi \cdot f_o \cdot t - \pi/2) \\ + B \cos(2\pi \cdot f_o \cdot t + \phi) \\ + 60 \sin(2\pi \cdot f_o \cdot t + \pi/4)$$

- Find $X_B = B e^{j\phi}$ such that $x(t) = 50 \cos(2\pi \cdot f_o \cdot t + \pi/4)$

- We observe that this is just a variation on the previous problem

- We start by writing

$$X = 50 e^{j\pi/4} \\ = 30 e^{-j\pi/2} + B e^{j\phi} + 60 e^{j(\pi/4 - \pi/2)}$$

so

$$X_B = B e^{j\phi} = 50 e^{j\pi/4} - [30 e^{-j\pi/2} + 60 e^{-j\pi/4}]$$

- Via a TI-89

TI-89 calculator screen showing the calculation of X_B . The display shows the expression $50 \cdot e^{i \cdot \pi / 4} - (30 \cdot e^{-i \cdot \pi / 2} + 60 \cdot e^{-i \cdot \pi / 4})$ and its rectangular form $-7.07107 + 107.782 \cdot i$. The bottom of the screen shows the function keys and page number 1/30.

TI-89 calculator screen showing the conversion of the complex number to polar form. The display shows the expression $50 \cdot e^{i \cdot \pi / 4} - (30 \cdot e^{-i \cdot \pi / 2} + 60 \cdot e^{-i \cdot \pi / 4})$ and its polar form $e^{1.63631 \cdot i} \cdot 108.013$. A red dashed box highlights the polar form. The bottom of the screen shows the function keys and page number 2/30.

$$B = 108.013$$

$$\phi = 1.636 \text{ rad}$$

- A lower level analysis can also be performed, as with #1

$$\begin{aligned} X_B &= 50 \left[\cos\left(\frac{\pi}{4}\right) + j \sin\left(\frac{\pi}{4}\right) \right] \\ &\quad - 30 \left[\cos\left(\frac{\pi}{2}\right) - j \sin\left(\frac{\pi}{2}\right) \right] - 60 \left[\cos\left(\frac{\pi}{4}\right) - j \sin\left(\frac{\pi}{4}\right) \right] \\ &= [35.36 + j35.36] + [j30] + [-42.43 + j43.43] \\ &= -7.07 + j107.78 = 108.013 e^{j1.6363} \end{aligned}$$

- The results agree