Sampling Example

Consider the signal

$$x(t) = 25\cos(2\pi(1000)t + \theta) + 10\cos(6000\pi t + \phi)$$

which is sampled at $f_s = 2500$ Hz to obtain $x[n] = x(n/f_s)$.

- Is there any aliasing present?

  - Note that the frequencies present in $x(t)$ are 1000 Hz and 3000 Hz, and since $f_s = 2500$ Hz, we see that 2500 is not greater than 3000
  - The 3000 Hz sinusoid will be aliased down to $3000 - 2500 = 500$ Hz

- What are the principle alias frequencies, in Hz, present in $x(t)$ relative to the given sampling frequency?

  - The principle alias band, in terms of positive frequencies, lies on the interval $[0, f_s/2)$ or [0, 1250) Hz
  - We have $f_1 = 1000 < 1250$ so this is a principle alias already
  - We have $f_2 = 3000 > 1250$, to find the principle alias we seek $f_{p2} = lf_s - f_2$ or $f_{p2} = f_2 - lf_s$ for l and integer such that $f_{p2} \in [0, 1250]$
  - The result is that for $l = 1$ $f_{p2} = 3000 - 2500 = 500$ Hz

- Find the corresponding $\hat{\omega}$ for each of the principle alias frequencies of $x(t)$

  - We convert from f in Hz to $\hat{\omega}$ in rad/sample using $\hat{\omega} = 2\pi f/f_s$
  - The frequencies of interest are

$$\hat{\omega}_1 = 2\pi\frac{1000}{2500} = 0.8\pi \qquad \hat{\omega}_{2p} = 2\pi\frac{500}{2500} = 0.4\pi$$

- Sketch the amplitude line spectra plot of $x(t)$ along with the principle alias spectral lines, to make it clear where aliasing is present