FIR System Function Example

- An LTI system has difference equation

$$y[n] = 2x[n] - 3x[n-1] + x[n-2]$$

- Find the system function $H(z)$
- Plot the poles and zeros of $H(z)$

- By z-transforming operationally both sides of the difference equation we have

$$Y(z) = 2X(z) - 3z^{-1}X(z) + z^{-2}X(z)$$

- So forming the ratio $Y(z)/X(z) = H(z)$ we have

$$H(z) = 2 - 3z^{-1} + z^{-2}$$

- To plot the poles and zeros we convert to positive powers of z

$$H(z) = \frac{2z^2 - 3z + 1}{z^2} = \frac{N(z)}{D(z)}$$

- The denominator roots (roots of $D(z)$) yields the system poles to be $z_{p1} = z_{p2} = 0$
- The numerator roots (roots of $N(z)$) can be found using the quadratic formula

$$z_{z1,2} = \frac{3 \pm \sqrt{9-8}}{2 \cdot 2} = \frac{3 \pm 1}{4} = 1, \frac{1}{2}$$

- Plotting in the z-plane we have