Frequency Response

Chapter 10

Chapter 9 dealt with time-domain response of continuous-time systems. The frequency response of continuous-time systems gives another view, just as it did for discrete-time systems in Chapter 6. The concept of frequency response is again motivated by applying a single sinusoid.

The Frequency Response Function for LTI Systems

- The output of an LTI system can be given in terms of the convolution integral

$$y(t) = h(t)*x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{10.1}$$

  where we recall that $h(t)$ is the (unit) impulse response of a system

- We choose to start the analysis with a single complex sinusoid

$$x(t) = Ae^{j\phi}e^{j\omega t}, -\infty < \omega < \infty \tag{10.2}$$

- The output is

$$\begin{aligned} y(t) &= h(t)*(Ae^{j\phi}e^{j\omega t}) \\ &= \int_{-\infty}^{\infty} h(\tau)Ae^{j\phi}e^{j\omega(t-\tau)}d\tau \\ &= \left(\int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}d\tau\right)Ae^{j\phi}e^{j\omega t} \\ &= H(j\omega)Ae^{j\phi}e^{j\omega t} \end{aligned} \tag{10.3}$$

- We have thus defined the frequency response of an LTI system as

Frequency Response

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}d\tau \tag{10.4}$$

<u>Example</u>: $h(t) = e^{-\alpha t}u(t)$

- From the definition

$$\begin{aligned} H(j\omega) &= \int_{-\infty}^{\infty} e^{-\alpha\tau}u(\tau)e^{-j\omega\tau}d\tau \\ &= \int_{0}^{\infty} e^{-(j\omega+\alpha)\tau}d\tau \\ &= \left.\frac{e^{-(j\omega+\alpha)\tau}}{-(j\omega+\alpha)}\right|_0^{\infty} = \frac{1}{\alpha+j\omega} \end{aligned}$$

- Given the frequency response we can now plot the magnitude and phase response just like was done for a discrete-time system

- A major distinction here is that the frequency axis runs from $-\infty$ to ∞
- We can use Matlab to do this using either a direct calculation or the function `freqs()`

```
>> help freqs
 FREQS Laplace-transform (s-domain) frequency response.
    H = FREQS(B,A,W) returns the complex frequency response vector H
    of the filter B/A:
                          nb-1          nb-2
             B(s)    b(1)s     +  b(2)s     + ... +  b(nb)
      H(s) = ---- = --------------------------------------
                          na-1          na-2
             A(s)    a(1)s     +  a(2)s     + ... +  a(na)

    given the numerator and denominator coefficients in vectors B and A.
    The frequency response is evaluated at the points specified in
    vector W (in rad/s).  The magnitude and phase can be graphed by
    calling FREQS(B,A,W) with no output arguments.

    [H,W] = FREQS(B,A) automatically picks a set of 200 frequencies W on
    which the frequency response is computed.  FREQS(B,A,N) picks N
    frequencies.
```

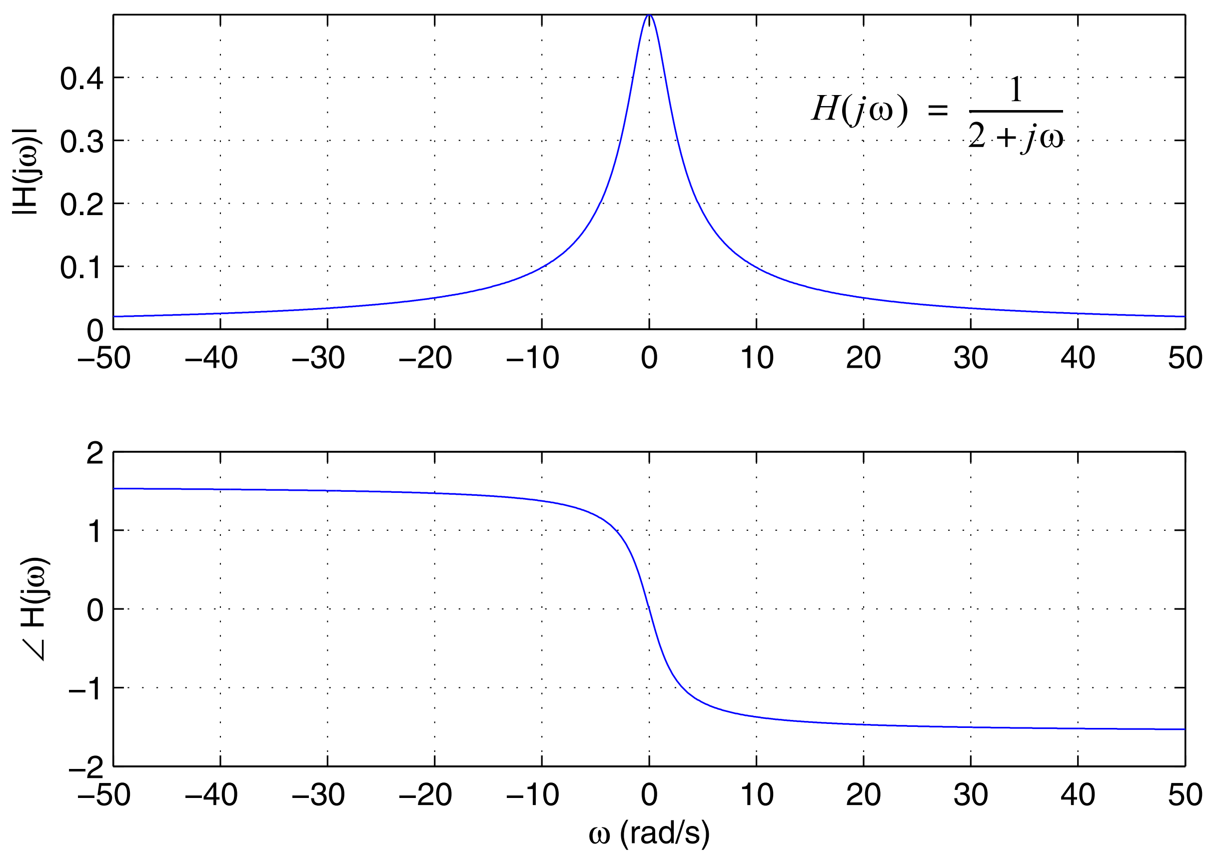
- The full understanding of `freqs()` is not possible until the Laplace transform is studied in systems and circuits
- The $H(j\omega)$ we have is of the form

$$H(j\omega) = H(s)\big|_{s = j\omega} = \left.\frac{1}{s+\alpha}\right|_{s = j\omega} = \frac{1}{j\omega + \alpha}$$

- Let $\alpha = 2$

```
>> w = -50:.1:50;
>> H = freqs([1],[1 2],w);
>> subplot(211)
>> plot(w,abs(H))
>> grid
>> ylabel('|H(j\omega)|')
>> subplot(212)
```

```
>> plot(w,angle(H))
>> grid
>> ylabel('\angle H(j\omega)')
>> xlabel('\omega (rad/s)')
```



Response to Real Sinusoid Signals

- The frequency response can also be used to find the system output when the input is a real sinusoid
- Just as in the case of discrete-time systems, when the input is

$$x(t) = A\cos(\omega_0 t + \phi), -\infty < t < \infty \tag{10.5}$$

the output is

$$y(t) = A|H(j\omega_0)|\cos(\omega_0 t + \phi + \angle H(j\omega_0)) \tag{10.6}$$

Symmetry of $H(j\omega)$

- Again utilizing what was learned for discrete-time systems, for a system having real impulse response the following symmetry condition holds

$$H(-j\omega) = [H(j\omega)]^* = H^*(j\omega) \tag{10.7}$$

which means that

Magnitude and Phase Symmetry

$$\begin{aligned} |H(-j\omega)| &= |H(j\omega)| \\ \angle H(-j\omega) &= -\angle H(j\omega) \end{aligned} \tag{10.8}$$

Response to a Sum of Sinusoids

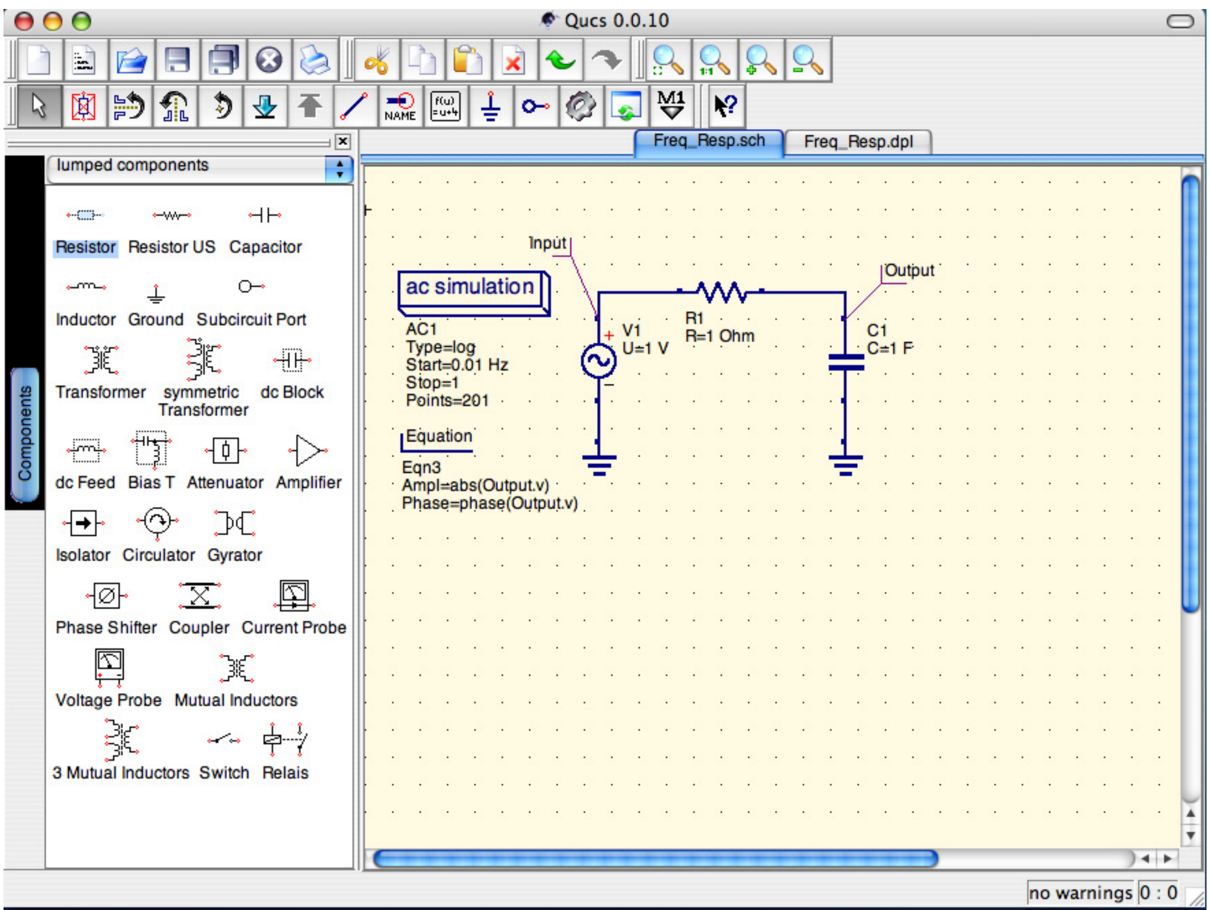
(10.9)

Periodic Signal Inputs

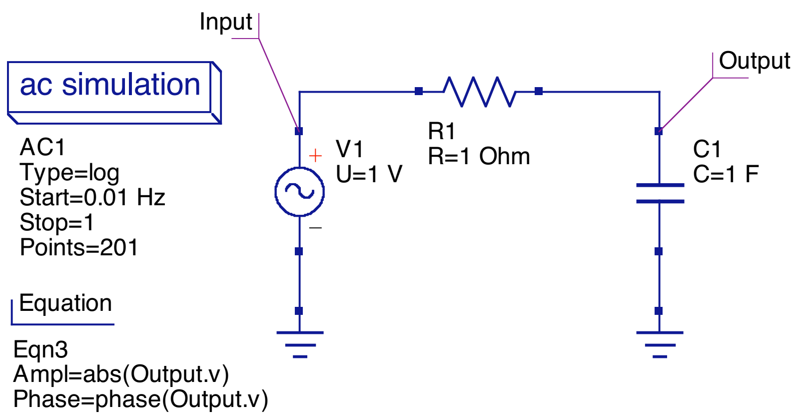
Ideal Filters

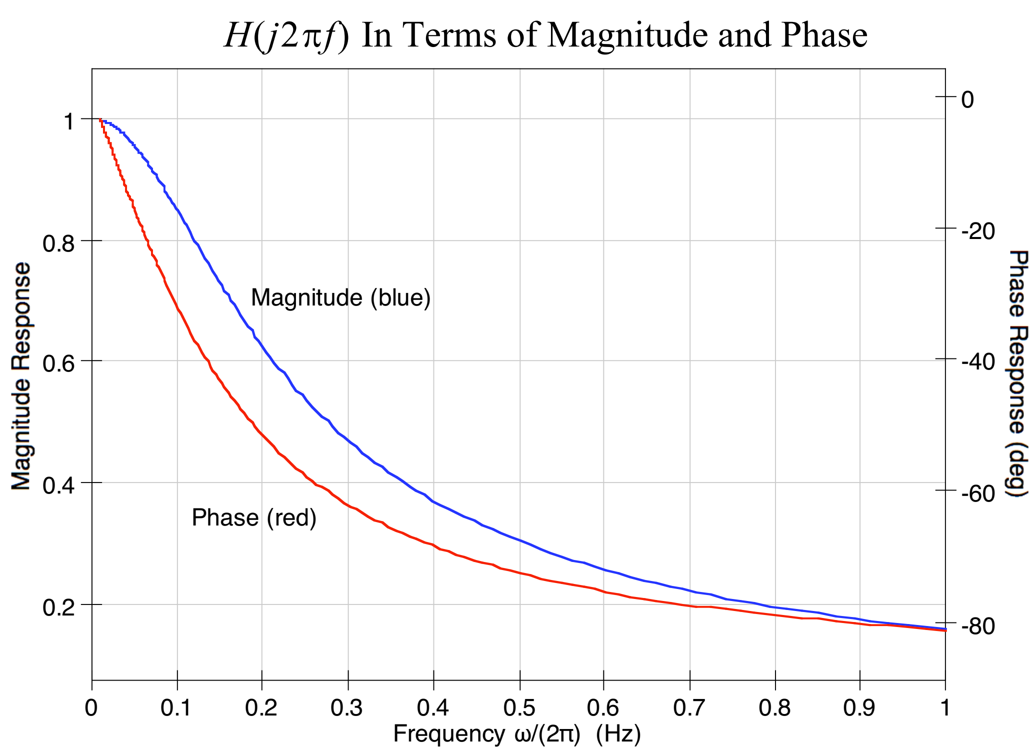
Simulation of Circuit Implementations

- As the study of systems and circuits moves forward in the courses ECE 2205, 3205, and ultimately electronic circuits, the need will eventually arise to simulate circuit based realizations of systems
- We now briefly introduce circuit analysis of a simple RC lowpass filter in terms of frequency response and time domain simulation for a pulse input and a sinusoid input
- A free cross platform compatible circuit simulator is Qucs, which stands for *quite a universal circuit simulator* (http://qucs.sourceforge.net/)

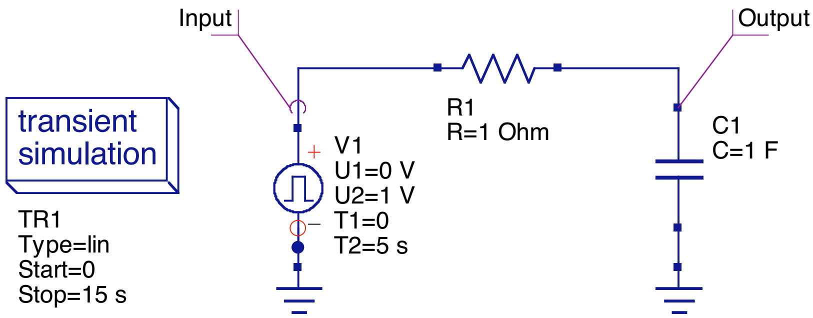


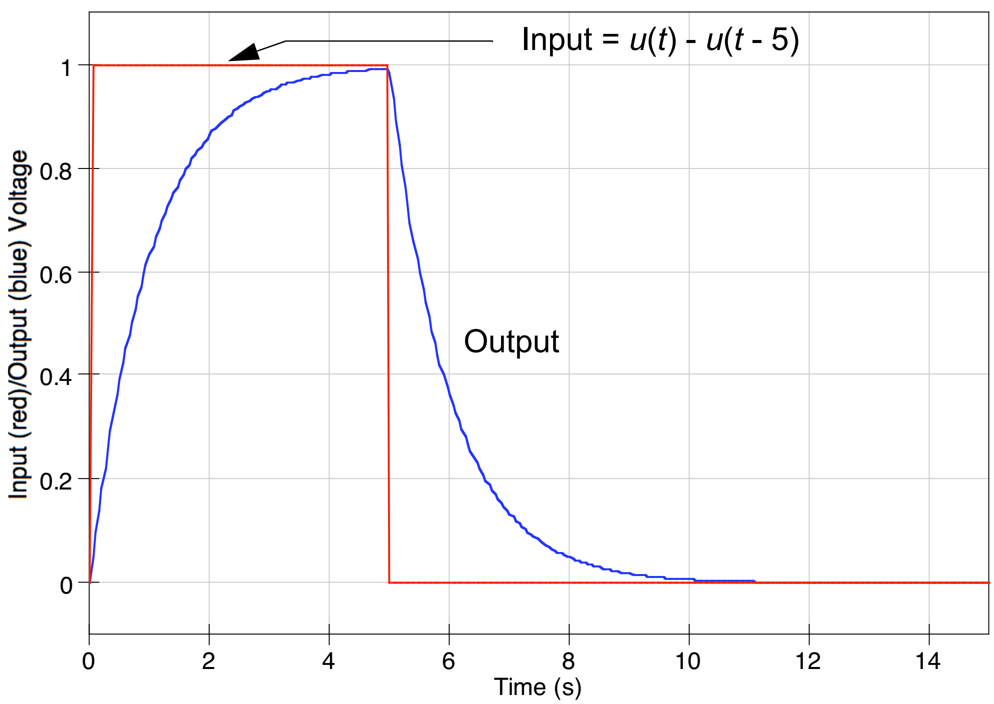
- *AC simulation* is the means for obtaining frequency response type results for a circuit implementation





- *Transient simulation* is used to obtain pulse input and sinusoidal input time-domain simulations





- Now consider the time domain response to a 1 Hz sinusoidal input applied starting at $t = 0$