System Modeling in Time and Frequency Domains Part II

Chapter 5

Introduction

- In this brief chapter I continue to develop system modeling concepts started in Chapter 3
- To be clear, electronic circuit design needs are motivated by system level requirements, that is a receiver design is required to *perform in such a way*
- At the system level design requirements can be given in terms of *time domain* and *frequency domain* characterizations
- The mathematics of system modeling gets quite complex, but is also very powerful[1]
- In this chapter the math will be very light, mostly pictures
- The idea is to motivate the needs/requirements of particular circuit designs

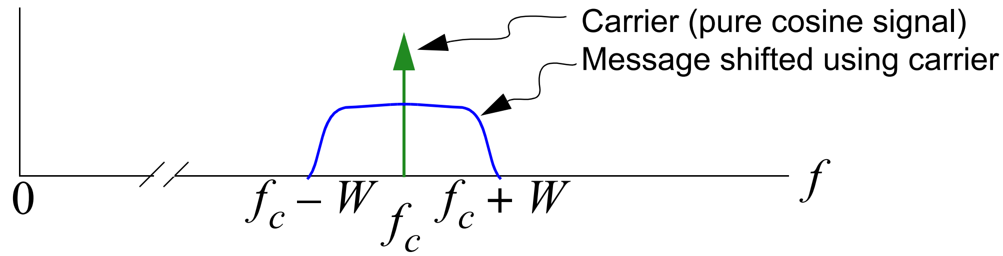
AM and FM Signal Modeling

Here I provide details on AM and FM signal generation, which by the way, is a capability of the Analog Discovery

1. M. Wickert, *Signals and Systems for Dummies*, Wiley, 2013.

Mathematical Details of Amplitude Modulation

- The spectrum of an AM signal is shown below:



- An amplitude modulated signal is formed by combining a *message signal* $m(t)$ together with a carrier signal $c(t)$

$$\begin{aligned} x_{\mathrm{AM}}(t) &= [A + m(t)]c(t) \\ &= A_c[1 + am_n(t)]\cos(2\pi f_c t) \end{aligned} \tag{5.1}$$

where

– $0 \le a \le 1$ defines what is known as the *modulation index*

– f_c is the carrier frequency in Hz, typically much greater than the highest frequency found in $m(t)$

– $m_n(t) = m(t)/|\min m(t)|$, which insures that the smallest value of $m_n(t)$ is -1

Example 5.1: Single Tone Modulation

- Let $m(t) = m_n(t) = \cos(2\pi f_m t)$ where $f_m = 5\,\mathrm{kHz}$ and set the carrier frequency to $f_c = 1\,\mathrm{MHz}$

- The waveform in mathematical terms is

$$x_{\mathrm{AM}}(t) = A_c[1 + a\cos(2\pi(5000)t)]\cos(2\pi \cdot 10^6 t) \tag{5.2}$$

- In communications signal processing the trig identity

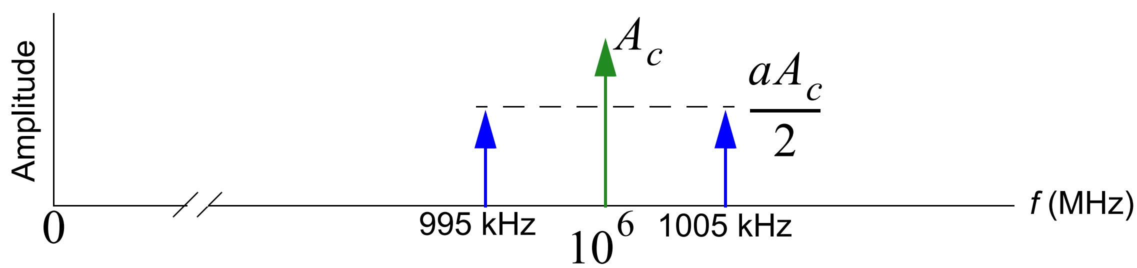
$$\cos(\theta)\cos(\phi) = \frac{1}{2}[\cos(\theta+\phi) + \cos(\theta-\phi)] \tag{5.3}$$

is used frequently!

- Making use of (5.3) to simplify (5.2) fully breaks down AM for a single tone message into three tones:

$$\begin{aligned} x_{\mathrm{AM}}(t) = \frac{A_c}{2}[&\cos(2\pi(10^6 + 5000)t) \\ &+ \cos(2\pi(10^6 - 5000)t)] \\ &+ A_c\cos(2\pi\cdot 10^6 t) \end{aligned} \tag{5.4}$$

- The single-sided theoretical spectrum thus takes the form:

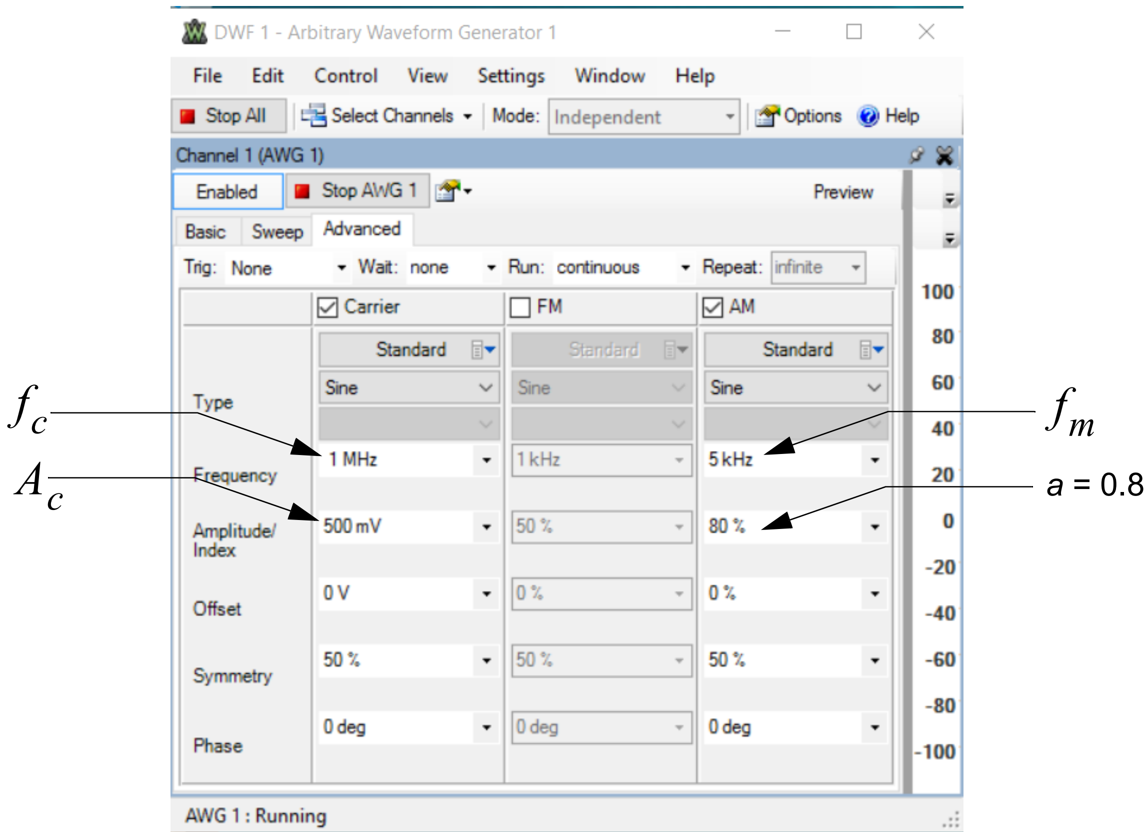


  – The center tone or spectral line is due to just the carrier signal

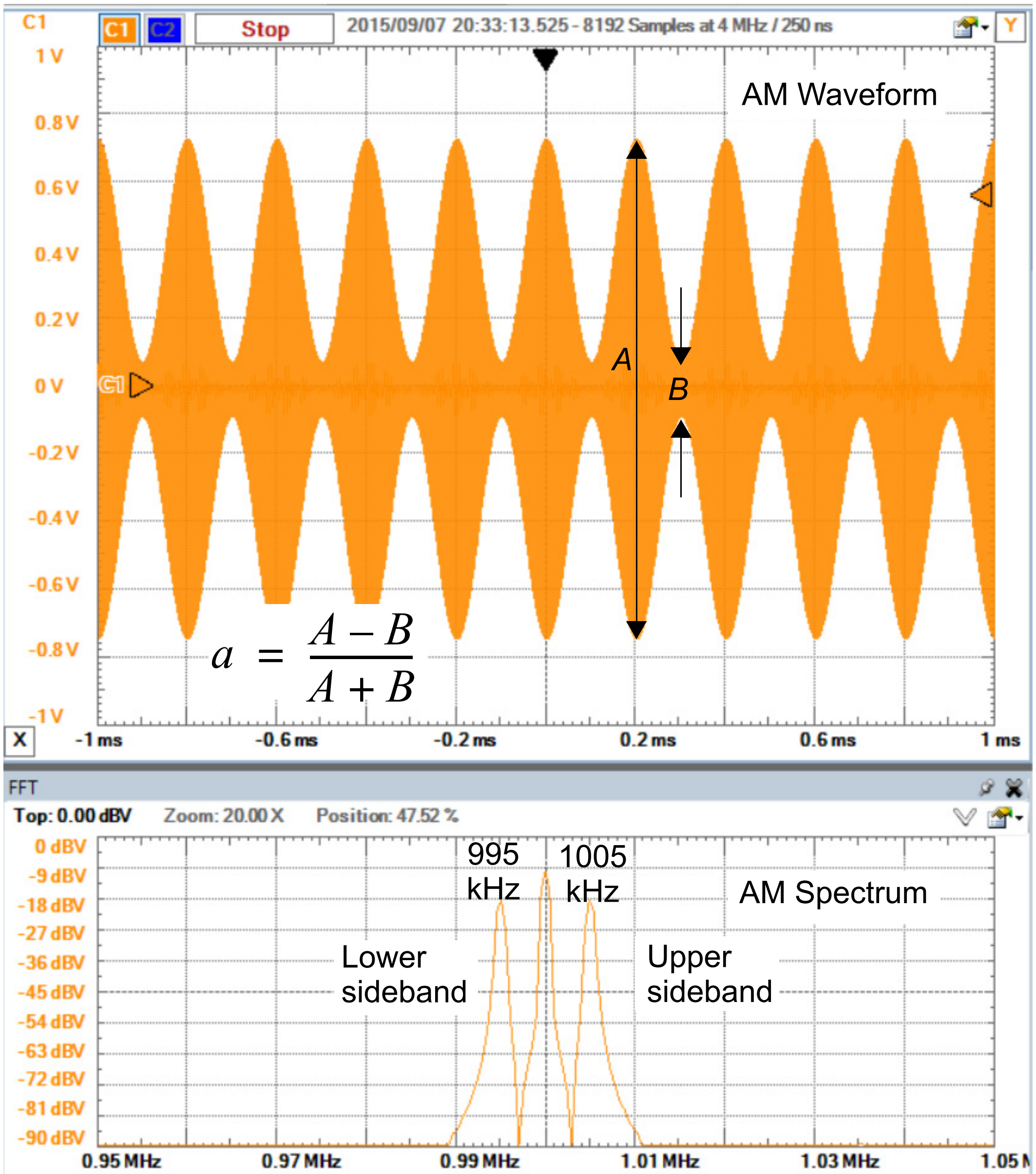
  – The spectral lines either side of the carrier are referred to as the *upper* (1005 kHz) and lower (995 kHz) sidebands

  – Further note that the sidebands are at least 6 dB below the carrier spectral line; why? {if $a = 1$ you have $20\log_{10}(2)$ as the factor}

- Using the Analog Discovery you can create this waveform using the Arbitrary Waveform Generator



- **Note**: There are additional parameters that may be set to create an AM waveform, but for this example the above noted values are of primary importance
- When viewed on the scope with the FFT spectrum analyzer turned on you observe the following:



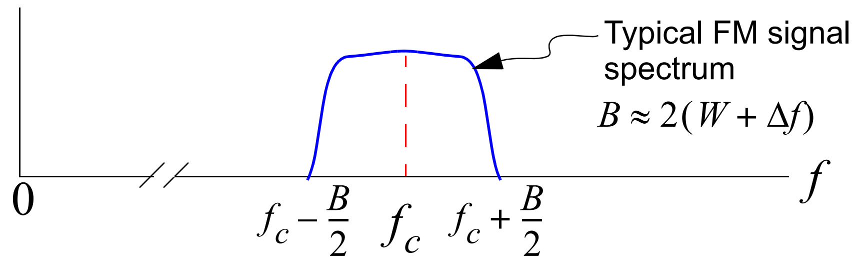
- **Note**: For the case of single tone modulation the modulation index, a, controls the relative strength of the upper and lower sidebands
  - The sidebands should be down

$$-20\log_{10}(0.8/2) = 7.96 \text{ dB} \tag{5.5}$$

  - The time domain peak-to-peak (A) and valley-to-valley (B) measurements also directly relate to a

Mathematical Details of Frequency Modulation

- The spectrum of an FM signal is shown below:



- An FM signal is formed by making the instantaneous frequency of the carrier sinusoidal signal vary with the amplitude of the message signal $m(t)$
- In mathematical terms this is represented as

$$x_{\mathrm{FM}}(t) = A_c \cos\left[2\pi f_c t + 2\pi f_d \int_{-\infty}^{t} m(\beta) d\beta\right] \tag{5.6}$$

  where
  - A_c is the carrier amplitude ($A_c^2/2$ is the power of the transmitted FM signal)
  - $m(t)$ is the message signal (just as in AM)
  - f_d is the frequency deviation constant (how much frequency variation for a given $m(t)$)
- For those with a calculus background, yes there is an integral in this expression
- The instantaneous frequency, $f_{\mathrm{inst}}(t)$, is $1/(2\pi)$ times the derivative of the cosine argument, making

$$\begin{aligned} f_{\text{inst}}(t) &= \frac{1}{2\pi} \cdot \frac{d}{dt}\left[2\pi f_c t + 2\pi f_d \int_{-\infty}^{t} m(\beta) d\beta\right] \\ &= f_c + f_d m(t) \end{aligned} \tag{5.7}$$

– The last line of (5.7) is what matters here!

– The instantaneous frequency of FM signal $x_{\text{FM}}(t)$ moves about f_c according to $f_d m(t)$

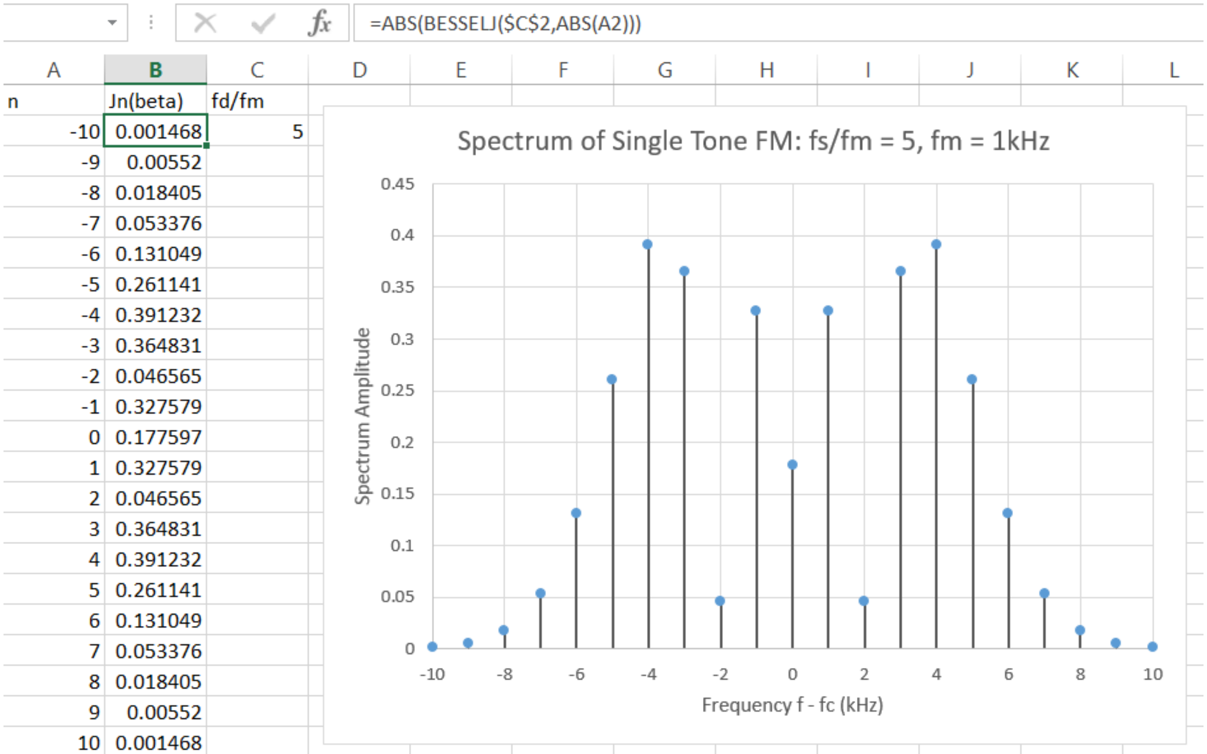
Example 5.2: Single Tone Modulation

- Let $m(t) = \cos(2\pi f_m t)$ where $f_m = 5\,\text{kHz}$ and set the carrier frequency to $f_c = 1\,\text{MHz}$
- The waveform in Mathematical terms is

$$\begin{aligned} x_{\text{FM}}(t) &= A_c \cos\left[2\pi \cdot 10^6 t + \frac{f_d}{f_m}\sin(2\pi f_m t)\right] \\ &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi f_c t + 2\pi n f_m) \end{aligned} \tag{5.8}$$

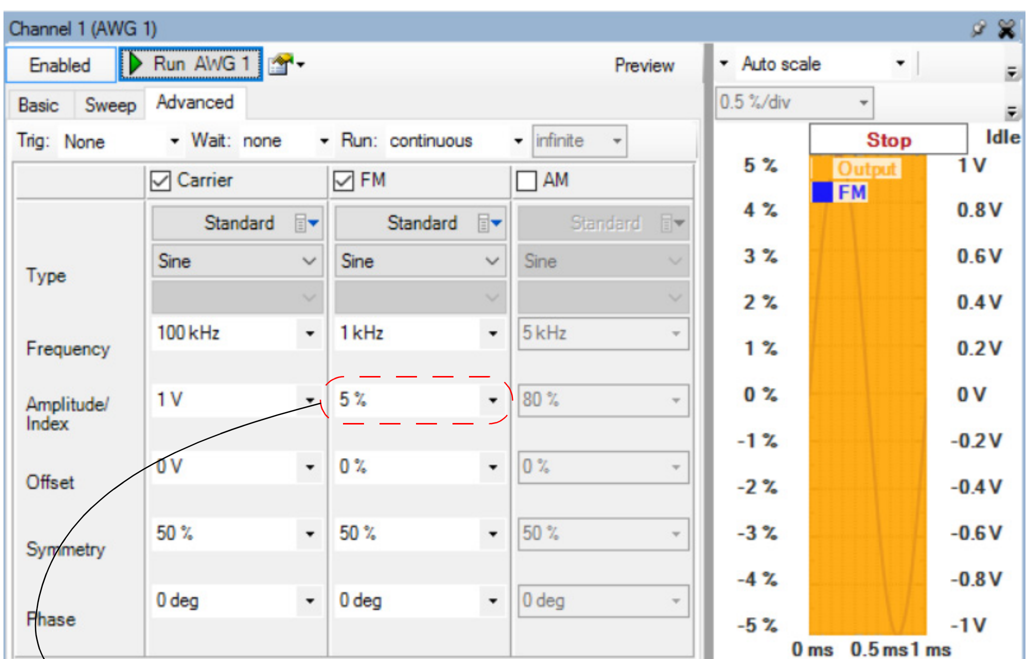
where $J_n(\beta)$ is a *Bessel function* (a special function in mathematics) and $\beta = f_d/f_m$ is known as the modulation index for single tone FM

- You can use Excel (or Python, Mathemtica, MATLAB) to make a spectrum plot



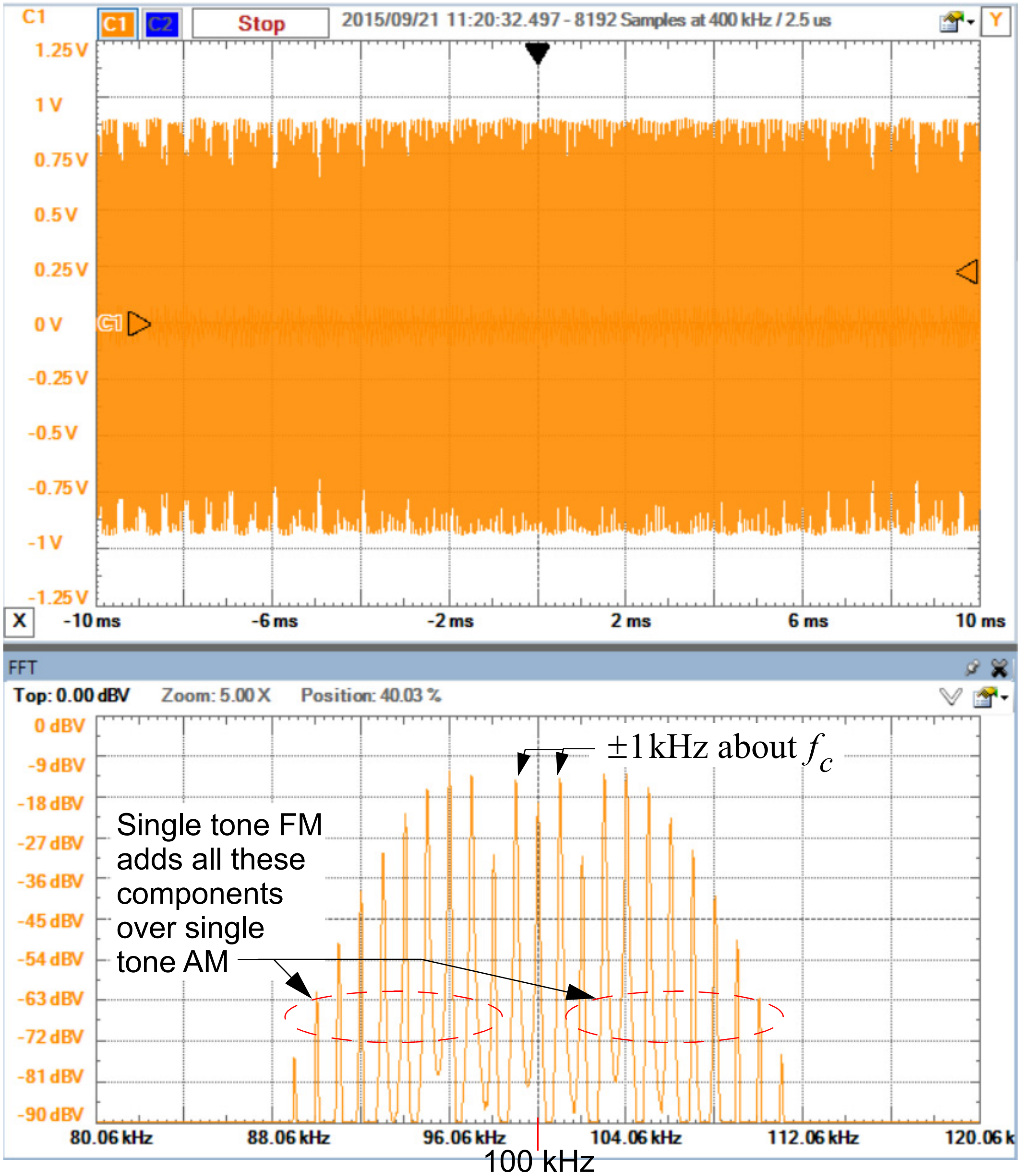
n	Jn(beta)	fd/fm
-10	0.001468	5
-9	0.00552	
-8	0.018405	
-7	0.053376	
-6	0.131049	
-5	0.261141	
-4	0.391232	
-3	0.364831	
-2	0.046565	
-1	0.327579	
0	0.177597	
1	0.327579	
2	0.046565	
3	0.364831	
4	0.391232	
5	0.261141	
6	0.131049	
7	0.053376	
8	0.018405	
9	0.00552	
10	0.001468	

- Using the Analog Discovery this waveform can be created using the AWG



The peak deviation is percent of the carrier frequency, here 100 kHz, so for fd/fm = 5 requires fd = 5 kHz or 5%

- Next observe the measured (FFT) spectrum using the scope



- The results look, taking into account the dB scaled used here versus the linear scale in the Excel plot

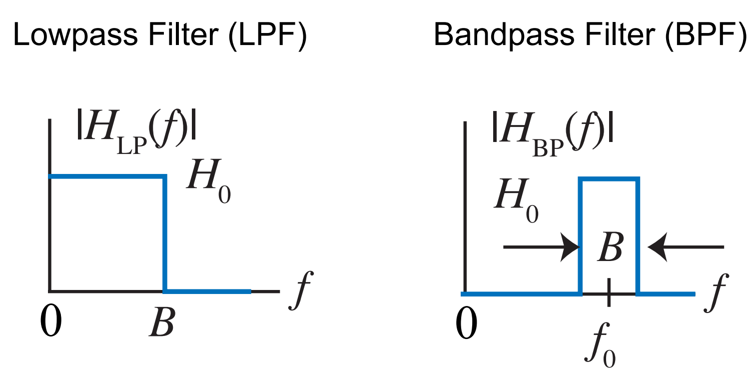
- The takeaway from this example is that with FM the occupied bandwidth easily exceeds the bandwidth of the message signal; the bandwidth is now about 2(1+5)k = 12 kHz!

System Modeling

- With signal time and frequency domain in hand, it is now time to take a look at some system modeling
- The goal is to look at simple system level modeling as it applies to radio receiver design
- Systems operate on one or more signal inputs to produce a new/modified output signal

Filters

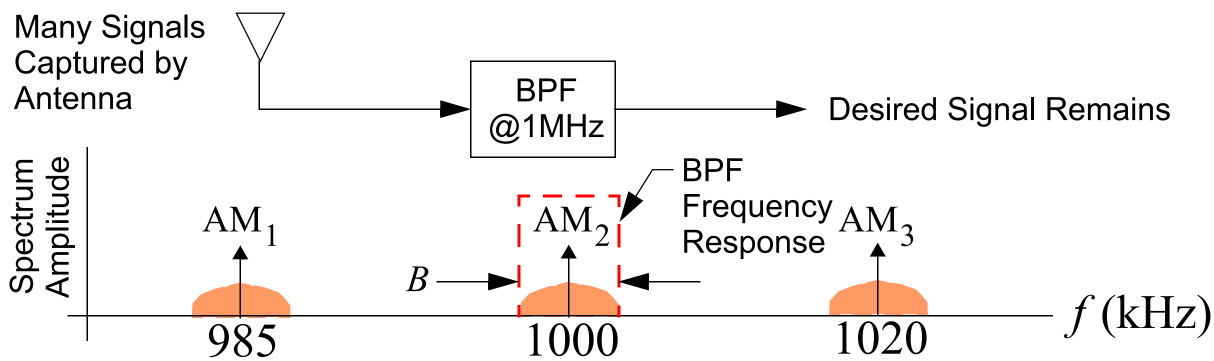
- A filter is a system that removes undesired signal content
  – Think of a coffee filter or water filter
  – The filter removes the unwanted and lets the wanted pass
- The filter *frequency response* of the filter is key concept here
- Consider ideal lowpass and bandpass filters:



- Both types are essential in receiver design
- When multiple received signals are present how do you select the desired signal? **Answer**: Use a filter!

Example 5.3: Dealing with Multiple Received Signals

- Consider the following received signals at the input to an AM radio receiver



- To receiver the signal at 985 kHz requires a re-tuning of the BPF from 1000 kHz to 985 kHz
- This design approach is the basis for the *tuned radio frequency* (TRF) AM receiver[1]
- In practice changing the center frequency of a filter and providing a narrow bandwidth B, is a design challenge
- There are design alternatives

- Using LTspice it is possible to create a behavioral model of both lowpass and bandpass filters
- The basic idea involves representing the filter mathematically as a ratio of polynomials in the s-domain
  - What is the s-domain?
  - For realizable filters, those that can be build using actual

1. https://en.wikipedia.org/wiki/Tuned_radio_frequency_receiver

circuit elements, the s-domain representation is the *system function* [1], $H(s)$, where

$$H(s) = \frac{b_M s^M + b_{M-1} s^{M-1} + \ldots + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \ldots + a_1 s + a_0} \tag{5.9}$$

N is the filter order, $M \leq N$, and s is a complex variable, meaning it has both real and imaginary parts (e.g., $s = \sigma + j2\pi f$

– The filter frequency response is obtained from the system function by letting $s = j2\pi f$, where yes $j = \sqrt{-1}$

- By using LTspice you can keep the detailed mathematics to a minimum
- Consider a popular class of filter known as *Butterworth*, having LPF system functions for $N = 1, 2, 3$ of the form

$$\begin{aligned} H_1^{\text{BU}}(s) &= \frac{\omega_c}{\omega_c + s}, \omega_c 2\pi f_c \\ H_2^{\text{BU}}(s) &= \frac{\omega_c^2}{s^2 + \sqrt{2}\omega_c s + \omega_c^2} \\ H_3^{\text{BU}}(s) &= \frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3} \end{aligned} \tag{5.10}$$

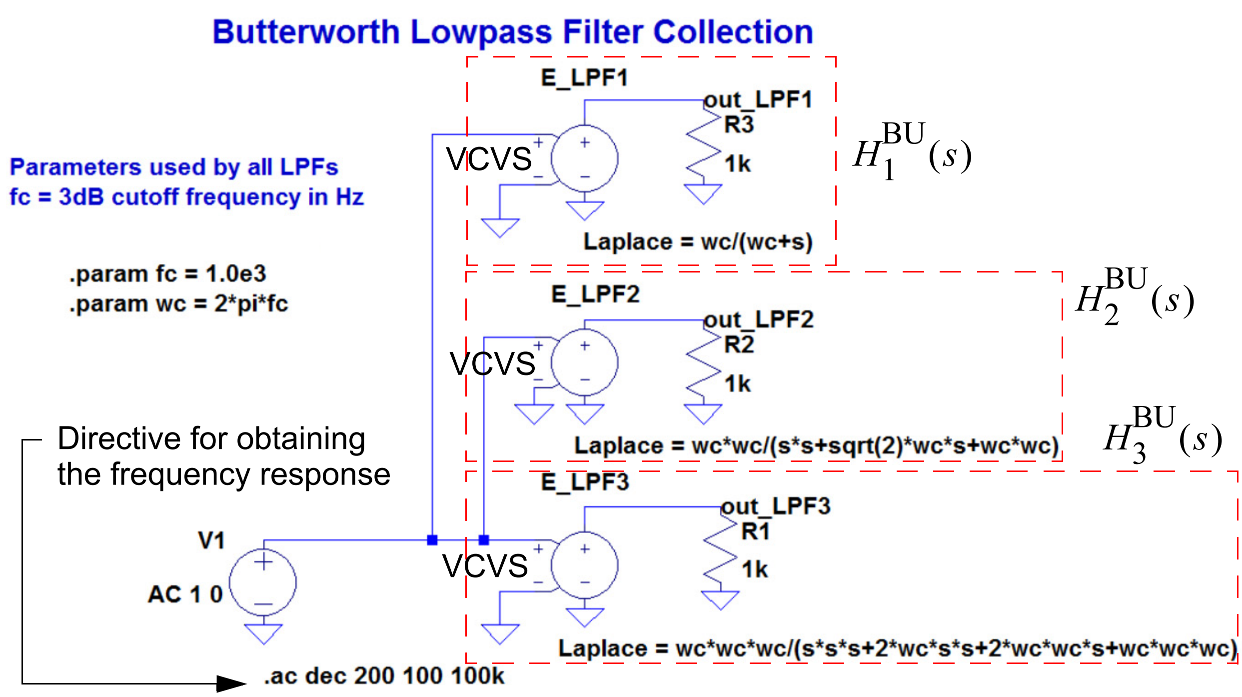
- The BPF version of the above is obtained by making the algebraic substitution

$$\frac{s}{\omega_c} \rightarrow \frac{\omega_o}{\Delta\omega}\left(\frac{s}{\omega_o} + \frac{\omega_o}{s}\right) = Q\left(\frac{s}{\omega_o} + \frac{\omega_o}{s}\right) \tag{5.11}$$

where ω_o is the filter center frequency, $\Delta\omega$ is the 3dB bandwidth of the filter in rad/s (divide by 2π to obtain Δf in Hz), and Q is the filter *quality factor*

Example 5.4: A Collection LPF Filters in LTspice

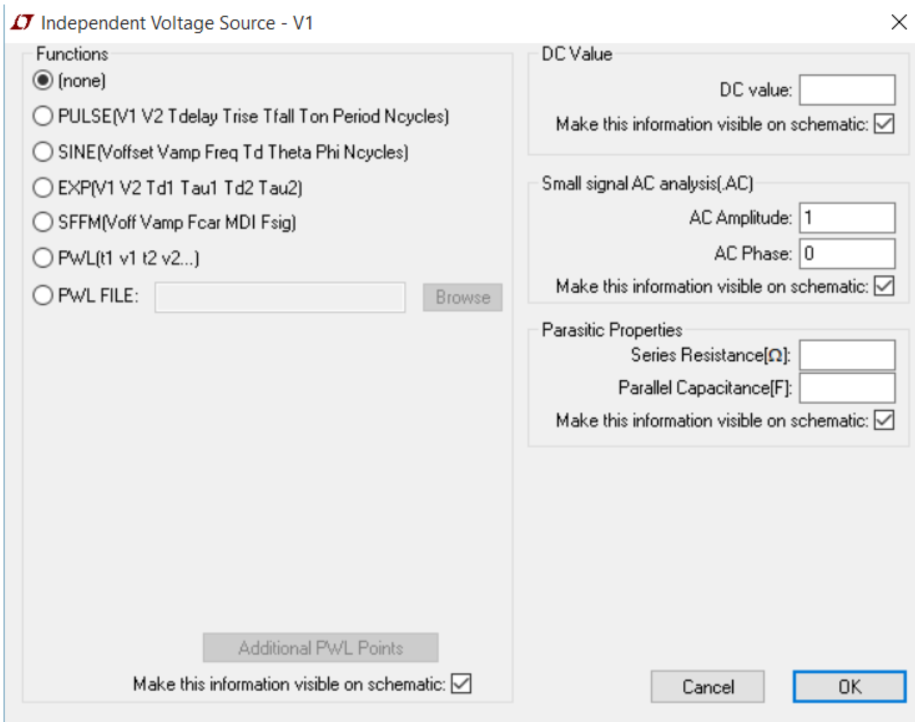
- The three lowpass filters of (5.10) are implemented inthe schematic shown below



- The key to direct implementation these filters is the voltage controlled voltage source (VCVS) and the *Laplace* statement found inside the VCVS model
  - Laplace comes from *Laplace transform*, which is an integral transform for transforming time domain signals and

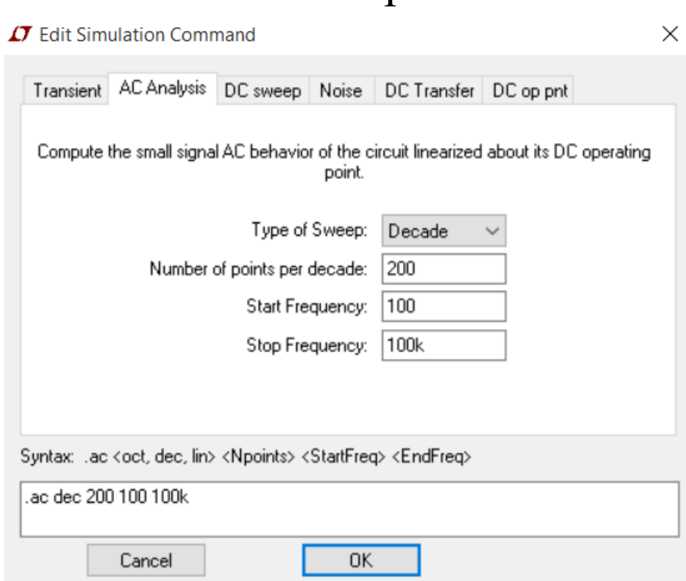
systems to their *s*-domain counter parts [1]

- To obtain the frequency response of a filter you replace the `.tran` directive with the `.ac` directive
- The details of setting an AC simulation involve two steps:
- **Step 1**: Set the AC signal source parameters



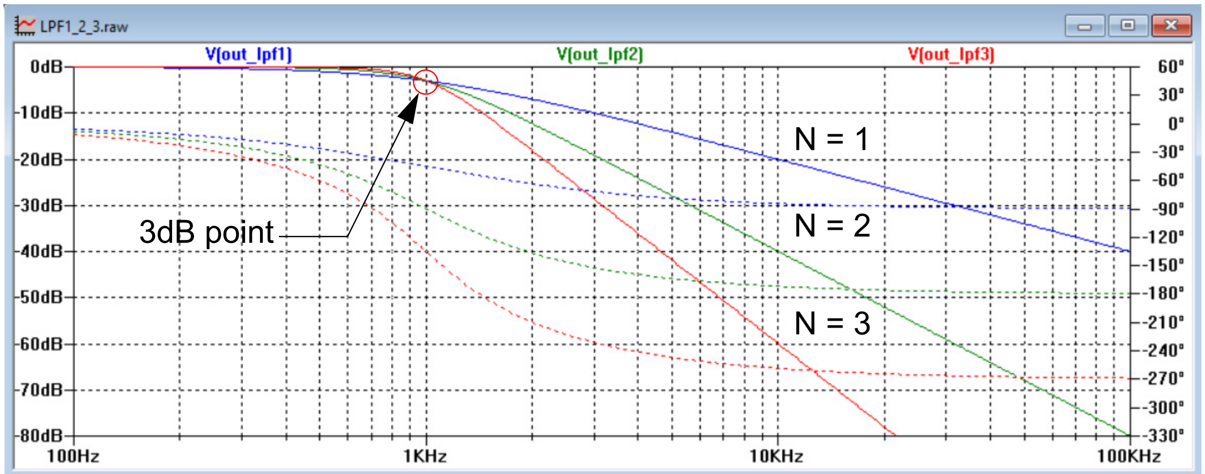
Set the AC source values

- **Step 2**: Set the AC simulation parameters



Final directive for schematic

- The frequency response plot is obtained after running the simulation



  – **Note**: Here $f_c = 1\,\text{kHz}$ for all three filters
  – The solid line is the filter voltage gain in dB and the dashed line (right axis) is the phase shift the filter introduces at a give frequency

- This LTspice example also introduces the directive `.param` which allows you schematic to contain variables

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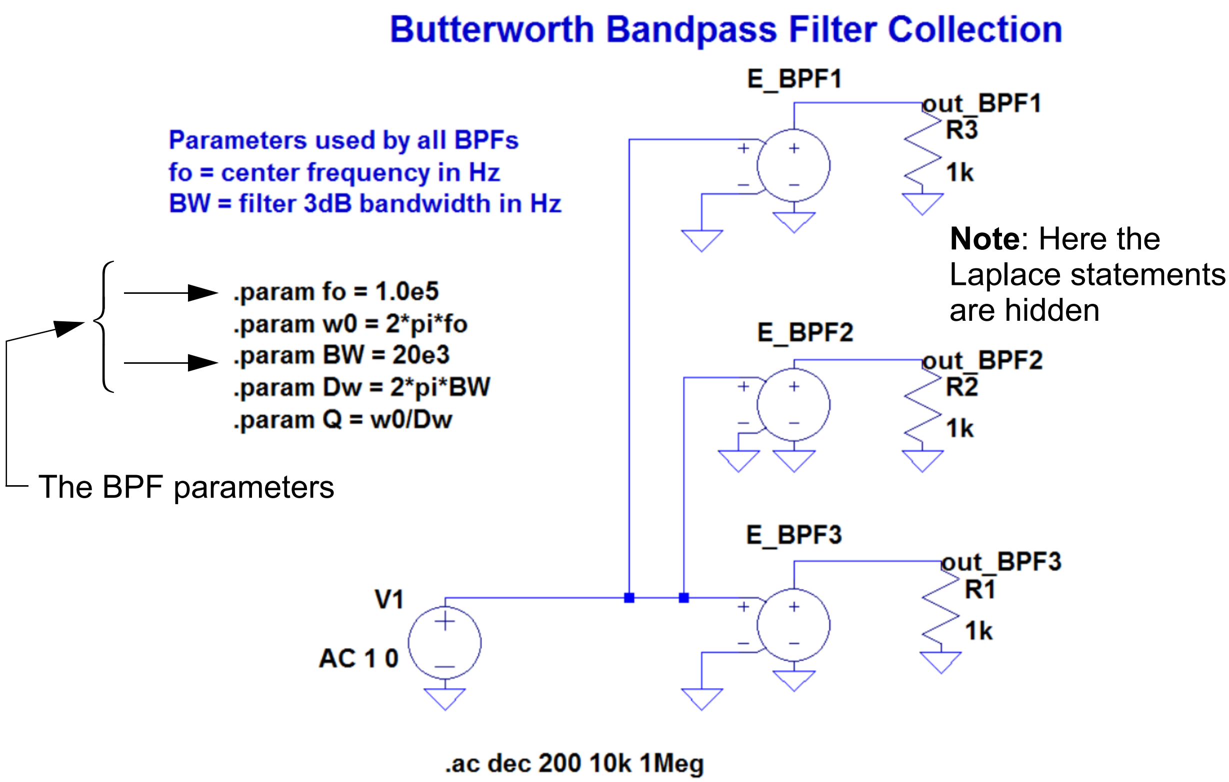
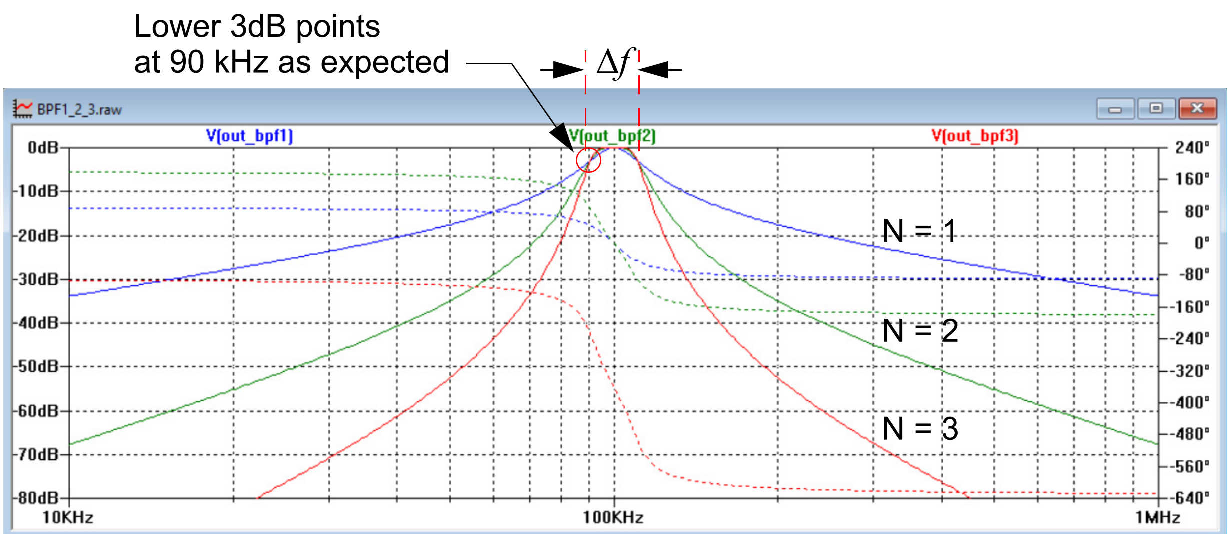
Example 5.5: A Collection of BPF Filters in LTspice

- This example is a rework of the LPF example using the transformation of (5.11)
- The controlling parameters reflect the attributes of a BPF, that is center frequency $\omega_o = 2\pi f_o$ and bandwidth $\Delta\omega = 2\pi\Delta f$



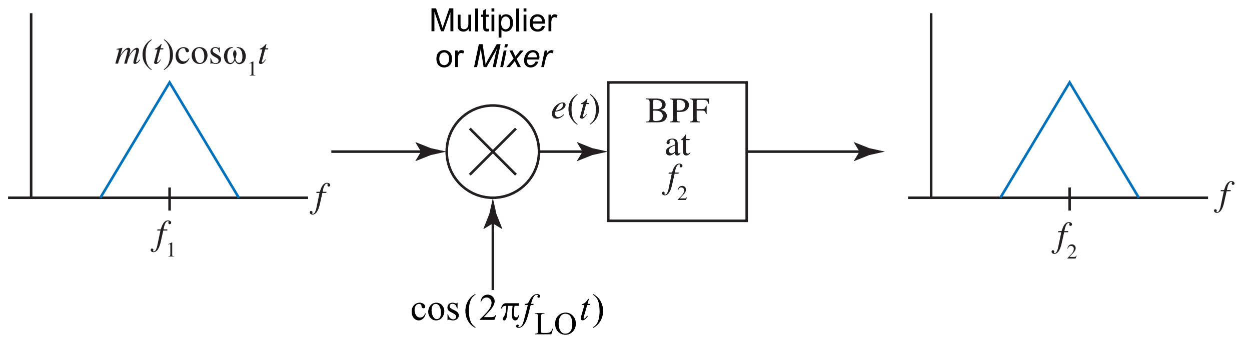
- The sweep range is different to accommodate the BPF centered at 100 kHz and bandwidth of 20 kHz



- Again both the magnitude in dB and phase is plotted
- Because of the transformation properties, the filter order is effectively doubled, i.e., $N = 2$ LPF implies $N = 4$ BPF

Frequency Translation and Mixing

- Frequency translation is moving signals from one center frequency to another



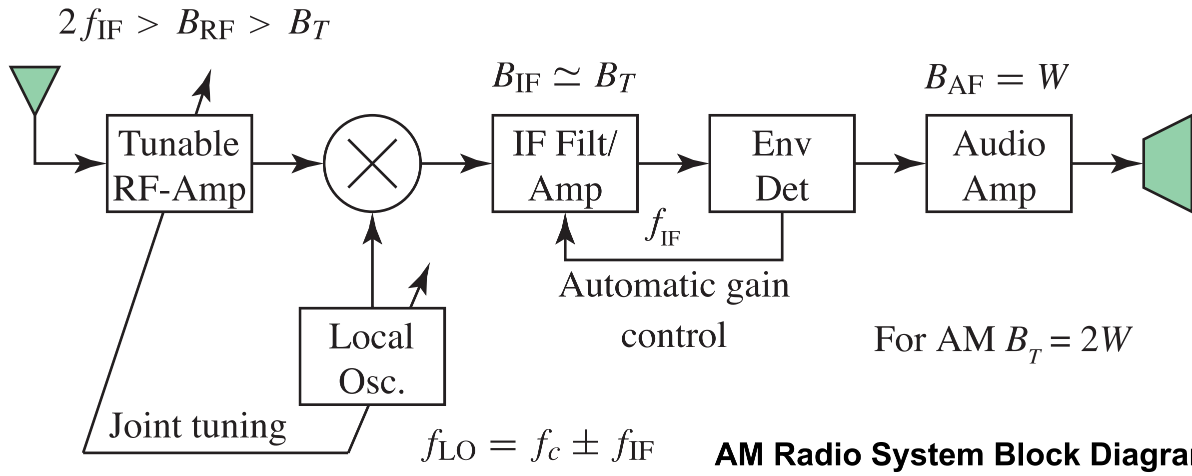
- To accomplish this you multiply (mix) the input signal with $\cos(2\pi f_{LO} t)$ to produce a new signal, $e(t)$
- The mixing action reduces down to the trig identity of (5.3), which says that $e(t)$ will consist of two signals, one centered at $|f_1 - f_{LO}|$ and the other centered at $|f_1 + f_{LO}|$
  – One of the two signals is retained and the other is rejected by using a BPF (see the block diagram)
- So how do you choose the local oscillator (LO) frequency f_{LO}?
  – The LO can sit above at $|f_1 + f_2|$ or below at $|f_1 - f_2|$

Example 5.6: Translation to 455 kHz

- In the design of an AM *superheterodyne* receiver[1] f_2 is referred to as the *intermediate frequency* (IF)
- In the AM radio kit you will assemble, $f_2 = f_{IF} = 455$ kHz

1. https://en.wikipedia.org/wiki/Superheterodyne_receiver

- To receive a signal at 1000 KHz you can choose f_{LO} to be either $1000 + 455 = 1455\,\text{kHz}$ or $1000 - 455 = 545\,\text{kHz}$

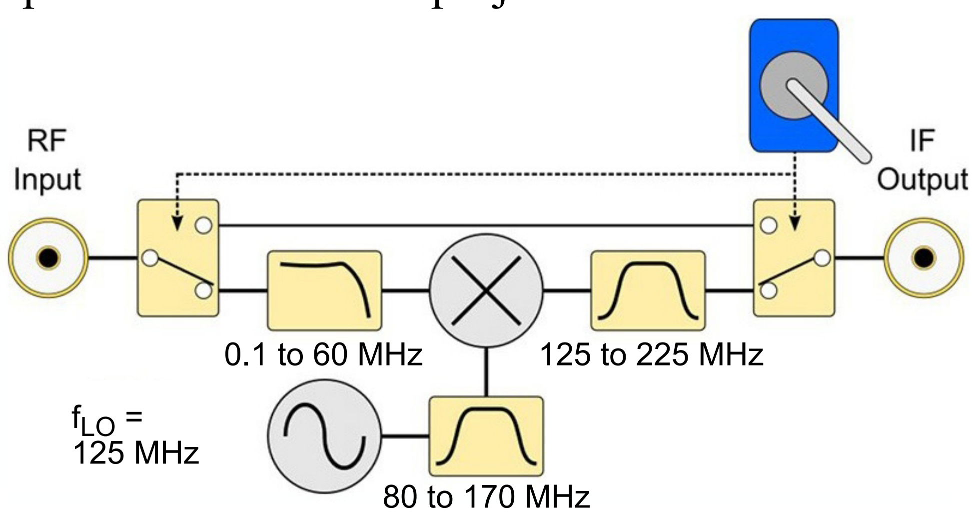


AM Radio System Block Diagram

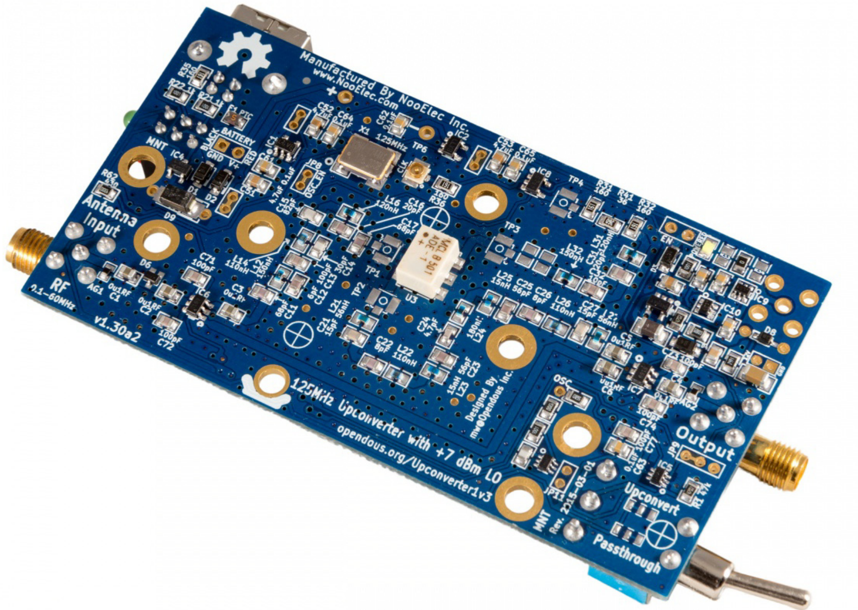
- The block diagram shows additional elements that are need to make a superhet receiver
  - Amplifiers (three different types shown)
  - Envelope detector

Example 5.7: Ham-It Up Upconverter

- An open source hardware project for the RTL-SDR

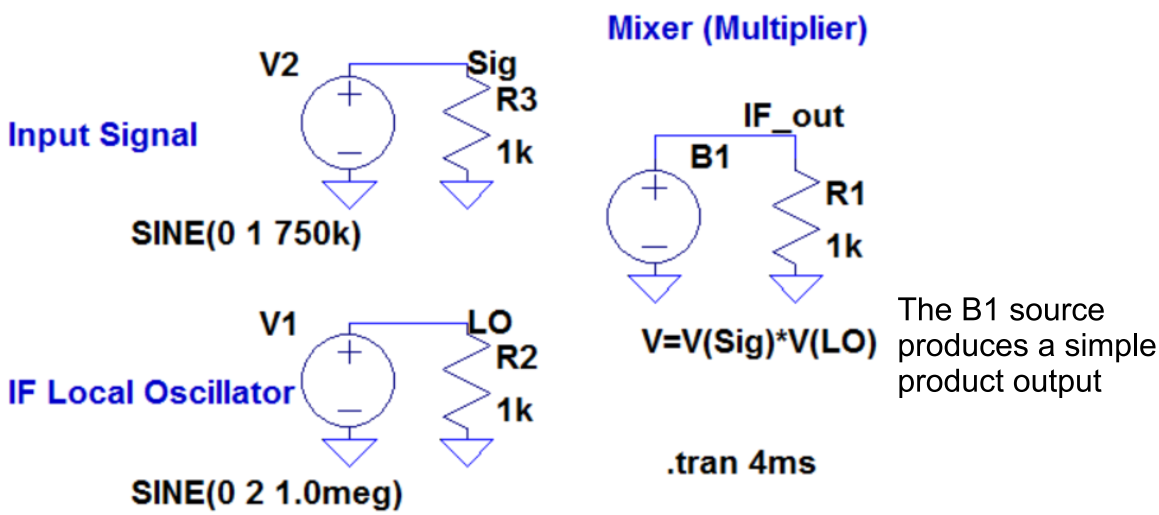


- The completed design:



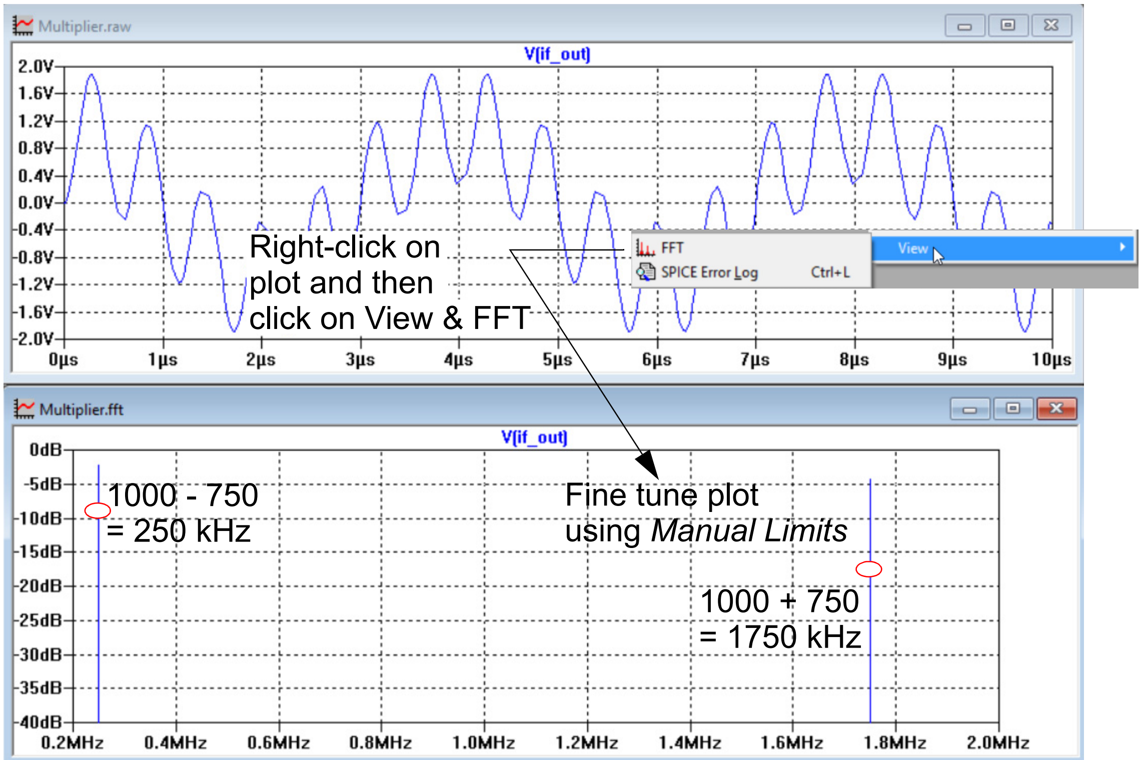
Example 5.8: Multipler in LTspice

- Again using LTspice behavioral modeling, in this case the `B-source`, you can process waveforms using mathematical functions, such as simply multiplying two signals together

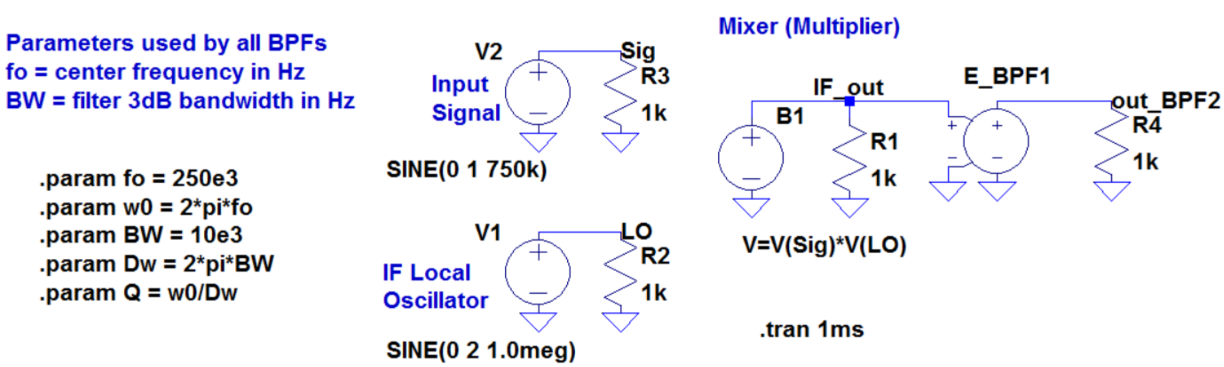


- Here the inputs are sinusoids at 750 kHz and 1000 KHz

- You expect the output to be sinusoids at $1000 \pm 750\,\text{kHz}$



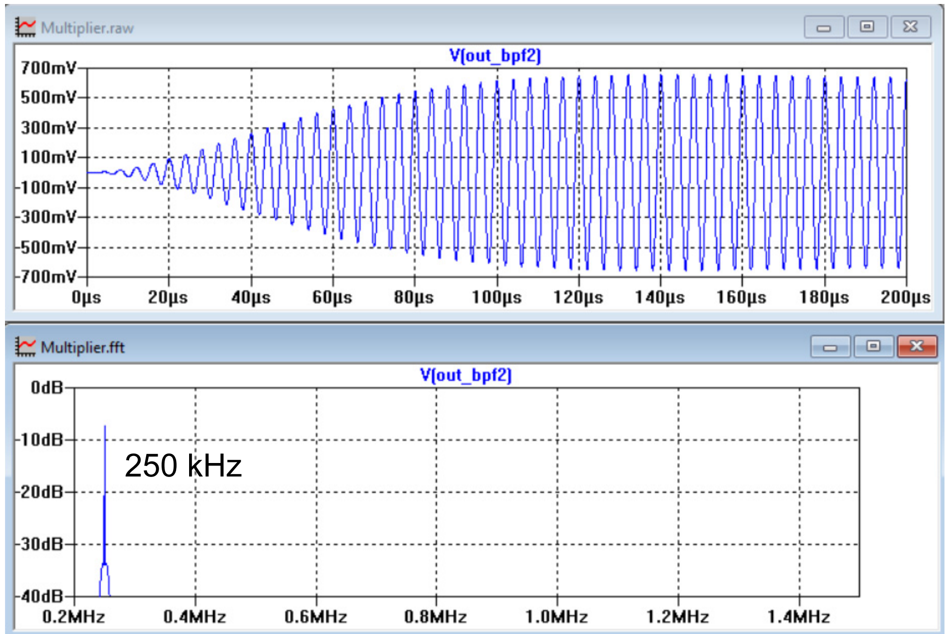
- If you add a BPF centered at 250 kHz to the out you expect just the 250 kHz signal to remain, right!



- To add the BPF I copy one filter (BPF2) from the Butterworth BPF collection schematic
  - **Warning**: The filter makes the simulation run very slowly
- I then set filter parameters accordingly, here I set $f_o = 250\,\text{kHz}$ and $\Delta f = 10\,\text{kHz}$

- The time and spectra at the BPF output

The filter charges up and produces a pure tone at 250 kHz



Gain Stages

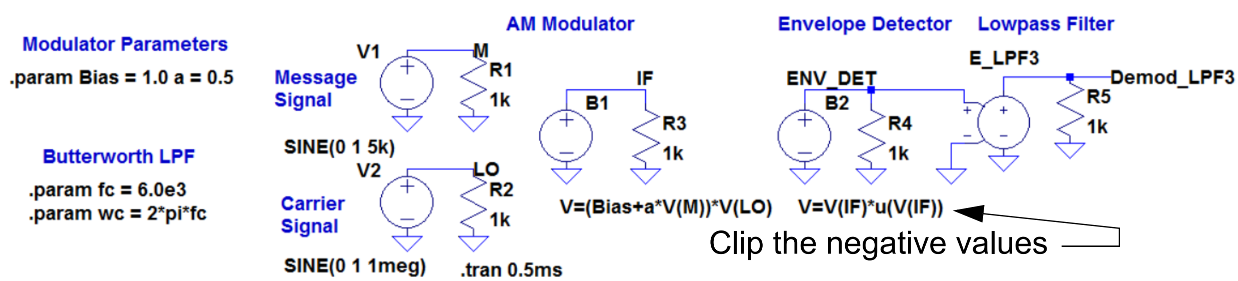
- The gain stages that you see in Example 5.4 are critical to the receiver design
- Amplifier circuits are needed
  – Radio frequency (RF) amplification at the receiver front end insure that the weak received signal is brought to a useful signal level
  – The IF stage of a receiver generally very high gain, but needs to be adjustable so that strong signals will not overload processing blocks downstream
  – The audio amplifier is needed to drive speakers or earbuds, which is not so much of a voltage gain issue, but the ability of the amplifier to drive a low resistance (impedance) load, e.g., 8 or 16 ohms

Demodulation

- A received and frequency translated radio signal needs to undergo *demodulation*, which is the undoing of the modulation found at the transmitter
  - Once the signal is demodulated it is ready to move to the audio amplifier
- It seems obvious that demodulation depends on the modulation type
  - AM demodulation is often accomplished with an *envelope detector*
  - Demodulation of FM requires a *frequency discriminator*

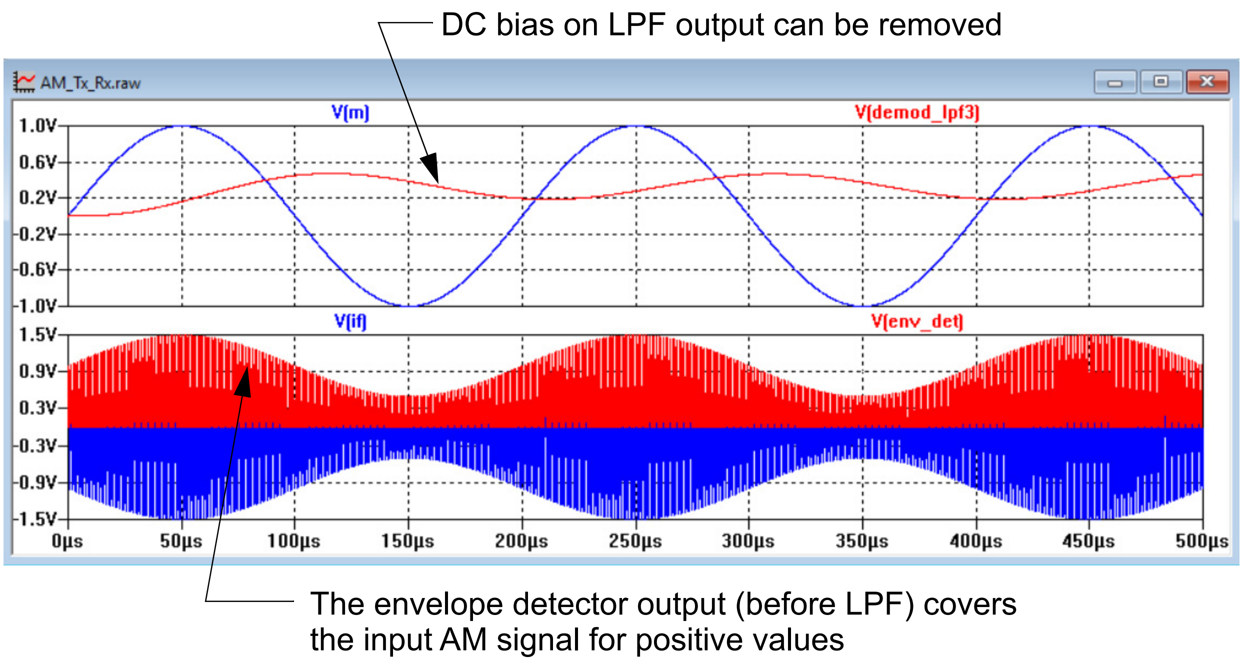
Example 5.9: LTspice Ideal Envelope Detector

- Using the B-source you simulate an ideal envelope detector
- To get started however a source of AM is needed and that can be created using another B-source model
- A lowpass filter thrown on the end of the processing chain will recover the original message (here a 5 kHz tone)

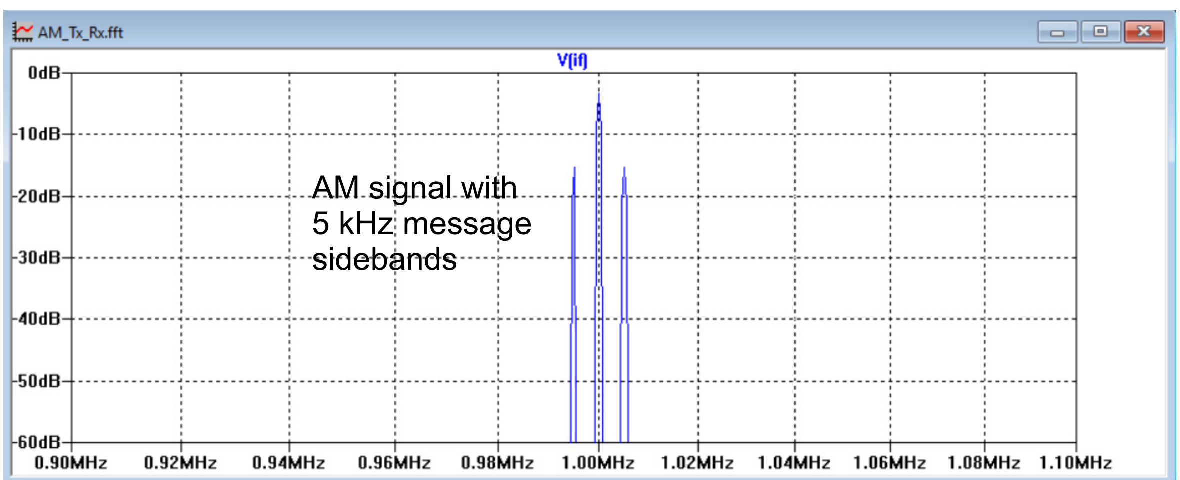


  - Note: `Pi = pi` is defined by LTspice
- The `ENV_DET` B-source returns the positive values or zero

- The outputs taken at four test points:

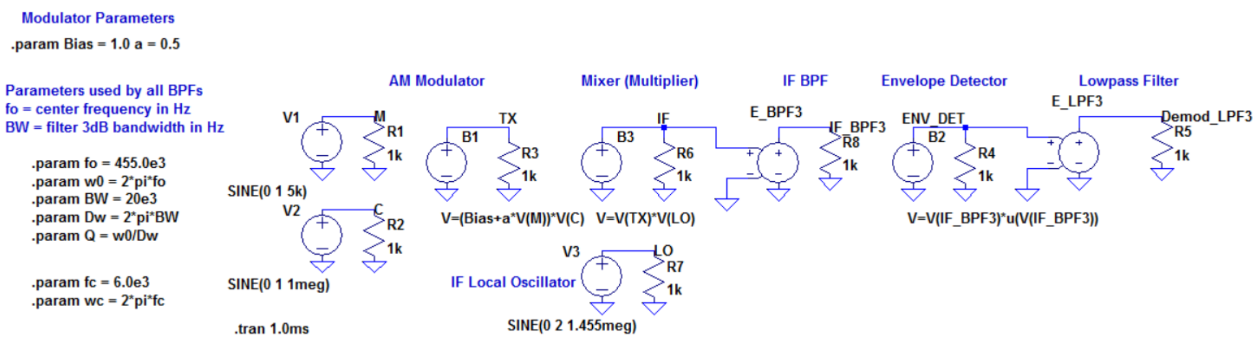


- Looking at the LTspice FFT spectra you should see upper and lower sidebands at 5 kHz relative to the 1 MHz carrier



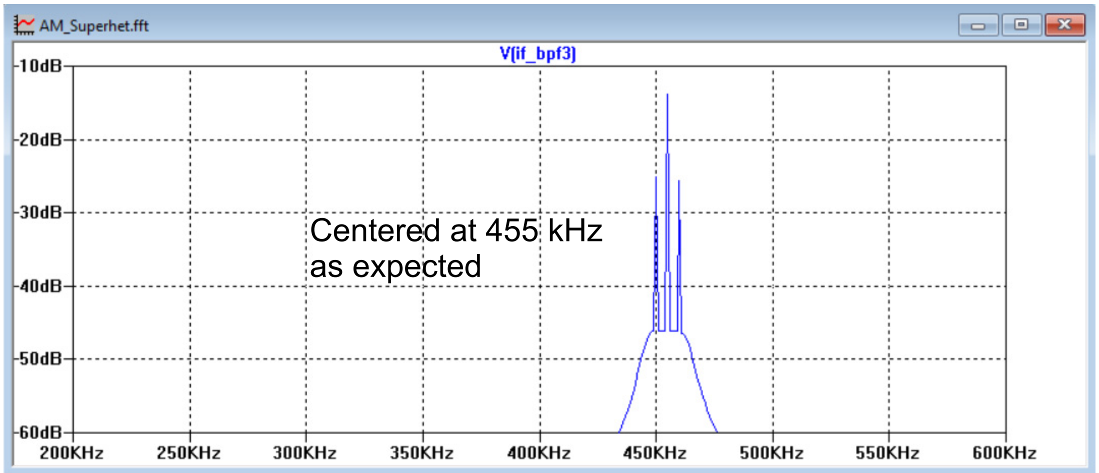
AM Superheterodyne Transceiver Simulation

- A complete AM transceiver system simulation can be build using LTspice

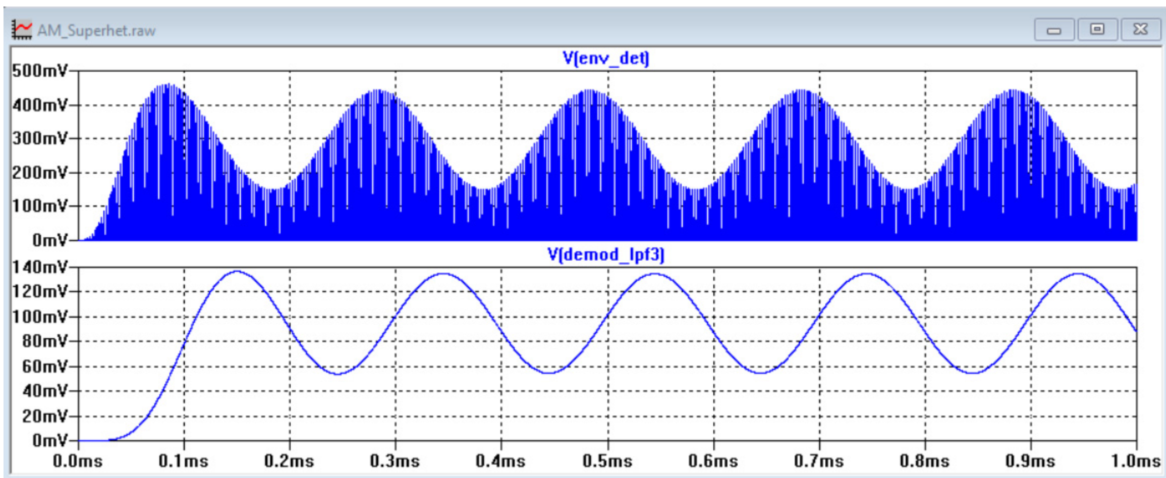


  - In the schematic you can see the carrier is at 1 MHz
  - The single tone message is at 5 kHz (modulation index $a = 0.5$)
  - A 455 kHz IF is chosen, so to receiver the 1 MHz AM signal high-side LO tuning is chosen, which implies $f_{LO} = 1.455\,\text{MHz}$
  - The IF BPF filter bandwidth is set to 20 kHz ($\pm 10\,\text{kHz}$)
  - The post envelope detector LPF cutoff is set to 6 kHz
- Output time domain waveforms and frequency domain spectra can be shown at various test points

- Consider first the AM spectrum at the IF filter output



- The envelope detector output, before and after the LPF



References

[1] M. Wickert, *Signals and Systems for Dummies*, Wiley, 2013.