Chapter 6

Electronic Circuit Design Using Capacitors and Inductors

Introduction

- The two remaining passive circuit elements of interest are the capacitor and inductor
- The common property between these elements is that they both store energy
  - The capacitor, C, stores energy in the electric field
  - The inductor, L, stores energy in the magnetic field
- The capacitor is composed of one or more metal plates separated by an dielectric insulator
- The inductor is composed of turns of wire, usually cylindrical
- In the s-domain (see Chapter 5) a modified form of Ohm's law is used to analyze circuits composed of RLC circuit elements
- Other circuit elements that remain to be covered include diodes and transistors and special purpose integrated circuits
- From here on out the circuit analysis will be more involved that simple resistor circuits

- I will be relying on the use of LTspice to demonstrate circuit behavior in an attempt to keep the mathematics from getting too detailed
- Two basic circuit behaviors of interest are:
  - *Transient conditions*, that is what happens in a circuit immediately following some change in signal or circuit configuration
  - *Steady-state conditions*, that is what is going on in the circuit when the applied signals and circuit configuration has been fixed for a relatively long period of time (time is realtive here, ms of μs might all the time needed to arrive at steady-state)

Applications

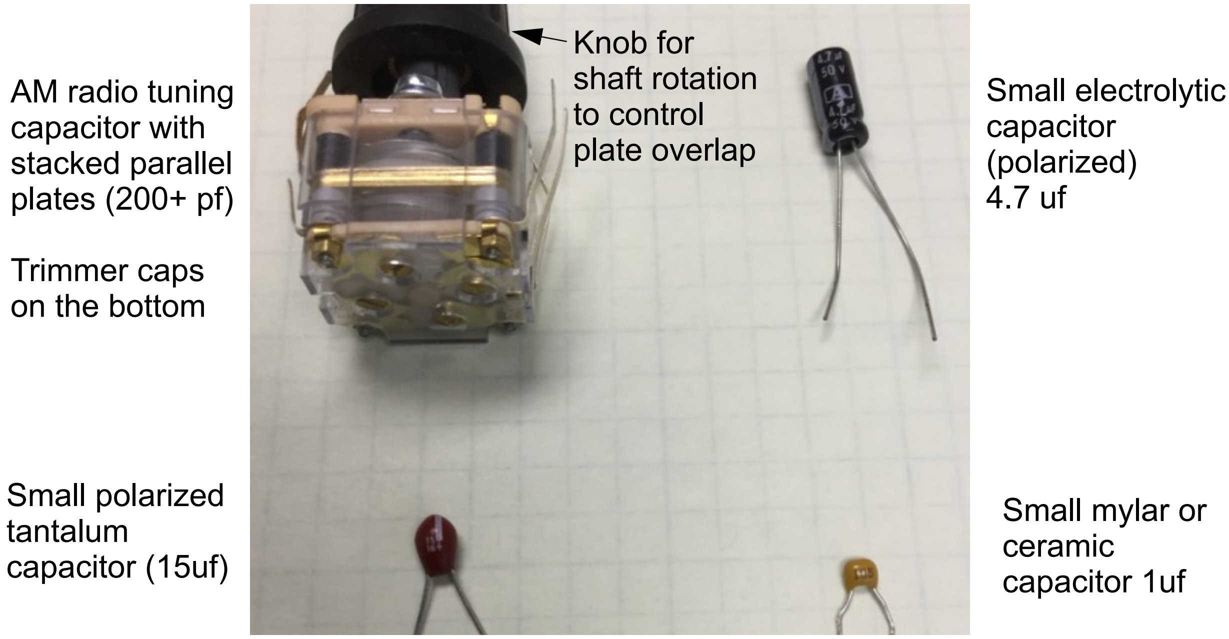
- Capacitors and inductors play a critical roles radio receiver circuits
- **Capacitors** fall into three applications areas:
  - **(1)** Form bandpass filters/resonators in RF and IF signal processing stages along with inductors; you will see this as an *LC tank circuit* role
  - **(2)** Block DC as signals are coupled from one amplifier block to the next; this is the *coupling capacitor* role
  - **(3)** Shut or bypass unwanted RF and IF signals to ground to provide signal integrity; this is the *bypass capacitor* role
  - **(4)** When combined with a resistor form a lowpass filter as

an envelope detector and in audio signal processing; this is the *RC lowpass filter* role

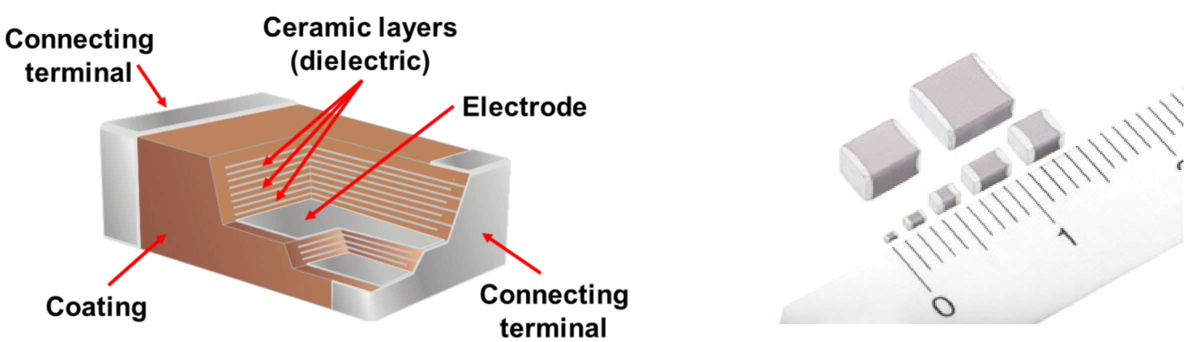
- **Inductors**:
  - **(1)** The LC tank circuit role discussed above
  - **(2)** The ferrite bar antenna coil forms a compact AM broadcasting antenna; *loopstick antenna* role
  - **(3)** In addition to the LC tank circuits in the IF amplifier chain of a superheterodyne receiver, IF transformers are used to couple stages (coupled coils by virtue of proximity) together avoiding the need for coupling capacitors in some cases; this is the IF amplifier *interstage coupling* role
  - **(4)** Use as a load impedance (reactance) in RF amplifiers that avoids the DC voltage drop of a load resistors; this is the *RF choke* role
- Beyond this course *lumped element* lowpass filters use capacitors
  - For example realization of the Butterworth system functions discussed in Chapter 5
  - Found in the Ham-It Up up-converter described in Chapter 5
- Lumped element bandpass filters (RF & IF) use both inductors and capacitors
  - Again the Butterworth BPF system functions of Chapter 5
  - Again, the Ham-It Up up-converter

Capacitors

- A circuit element that can store energy in the electric field between its two conductors separated by an insulator

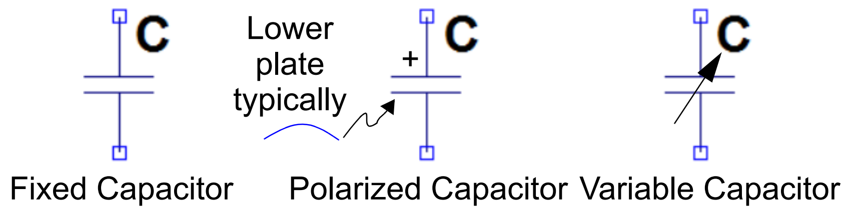


  – The capacitance value, C, has units of Farads (charge/V)
  – In modern electronics the chip capacitor is very common and offers very small size through a construction using many layers (parallel plates):

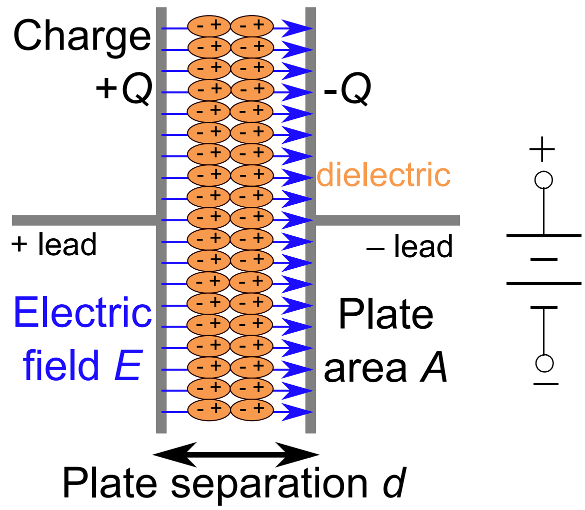


Multilayer ceramic capacitor (MLCC)

- Schematic symbols:



- Basic principles[1]
  - Connect a battery across the terminals of the capacitor and positive and negative charge distributes as shown to the right
  - The capacitance is proportional to A/d



- The dielectric filling material factors into the capacitance as well, making

$$C = \frac{Q}{V} = \varepsilon\frac{A}{d} = \varepsilon_0\varepsilon_r\frac{A}{d} \qquad (6.1)$$

where ε_0 is the permittivity of free space, 8.85×10^{-12} F/m, and ε_r is the relative permittivity or dielectric constant

- The energy stored in a capacitor is

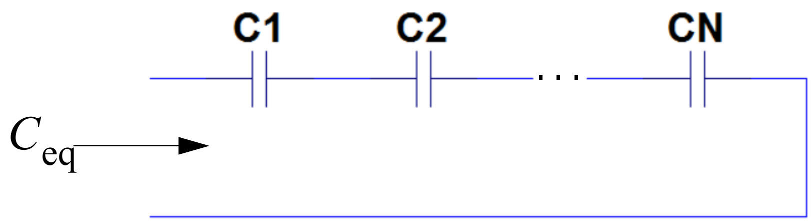
$$W = \frac{1}{2}CV^2 \text{ J} \qquad (6.2)$$

with V being the voltage across the plates

1. https://en.wikipedia.org/wiki/Capacitor

Parallel and Series Connections

- The formulas for equivalent capacitance when capacitors are placed in **series** is like the parallel formula for resistors

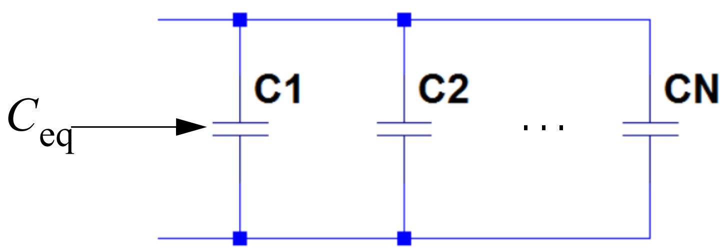


$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_M}} \tag{6.3}$$

- For the special case of just two capacitors is series is just

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \tag{6.4}$$

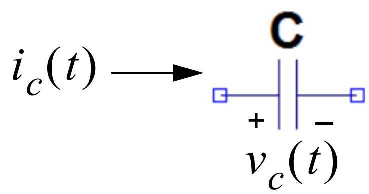
- The formulas for equivalent capacitance when capacitors are placed in **parallel** is the opposite as for resistors



$$C_{eq} = C_1 \| C_2 \| \cdots \| C_N = C_1 + C_2 + \cdots + C_N \tag{6.5}$$

Time Domain (Transient) Behavior

- The behavior of a capacitor in a circuit is governed by the voltage across the capacitor, the current through the capacitor, and of course the capacitance C



- The fundamental terminal relationship involves calculus:

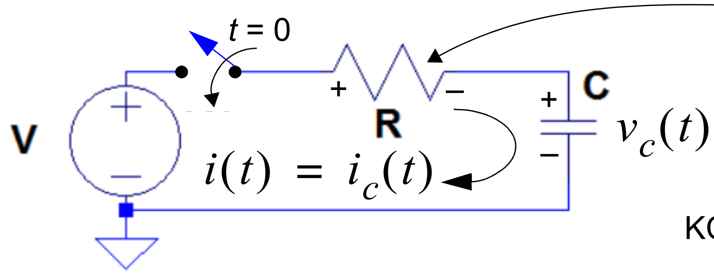
$$i_c(t) = C\frac{dv_c(t)}{dt} \approx C\frac{v_c(t) - v_c(t - \Delta t)}{t - (t - \Delta t)} \tag{6.6}$$

or in words, the current through a capacitor is C times the derivative of the voltage across the capacitor (slope of the voltage across the capacitor with respect to time)

- Using integration (from Calculus) you can solve for the voltage from the current plus the initial voltage across the capacitor

$$v_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c(\alpha)d\alpha + v_c(t_0) \tag{6.7}$$

- When a capacitor is placed in a circuit with resistors and other capacitors (maybe inductors too), you can use KCL and KVL (see notes Chapter 3) to solve for a voltage or current of interest



Note:

$$v_R(t) = Ri(t) = Ri_c(t)$$

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$\text{KCL} \Rightarrow V = RC\frac{dv_c(t)}{dt} + v_c(t)$$

- This is not that easy to solve (more so in the general case), as the pure algebra of resistor circuits becomes differential equations! (in the above figure a 1st-order diff-eqn)
- For first-order circuits we can use the so-called *inspection method* [2]
- **Good news**: LTspice works well for complex cases!

• **Inspection Method**: For first-order equations involving one capacitor (or one inductor) and resistors, the voltage or current of interest in response to a constant input applied at $t > 0$ is

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}, t \geq 0 \tag{6.8}$$

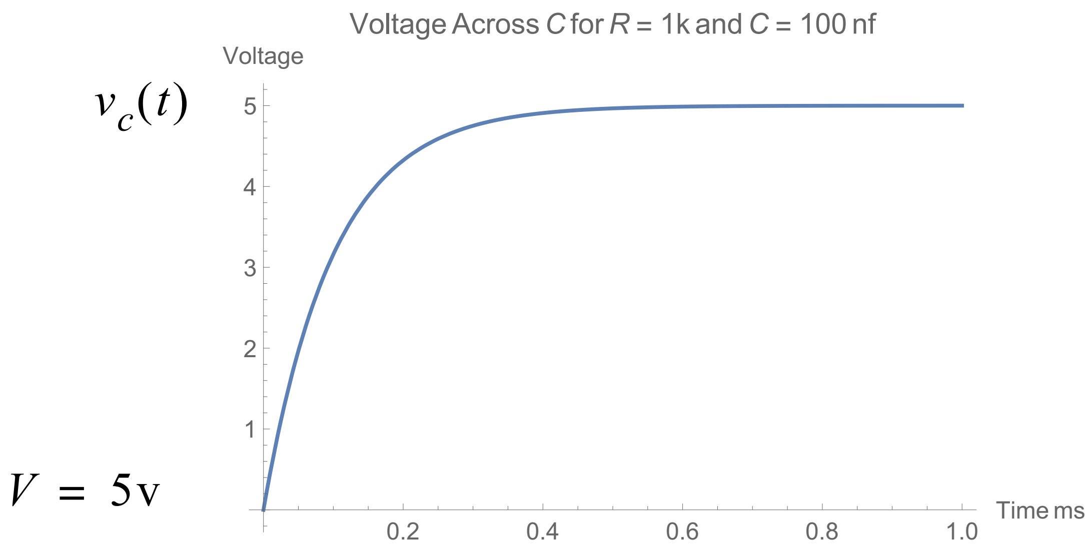
where τ is the circuit *time constant*

• Capacitor behavior under the inspection method:
  - The voltage across a capacitor cannot change instantly; it must charge up over time
  - At $t = 0$ the voltage across a capacitor is $v_c(0)$
  - At $t = \infty$ you find $v_c(\infty)$ assuming the capacitor is fully charged and the current flow is zero, making the capacitor appear as an open circuit (capacitor not present in the circuit anymore)
  - The time constant $\tau = R_{\text{eq}}C$, where R_{eq} is the equivalent resistance in series with the capacitor

• Consider now the circuit above and find the voltage $v_c(t)$

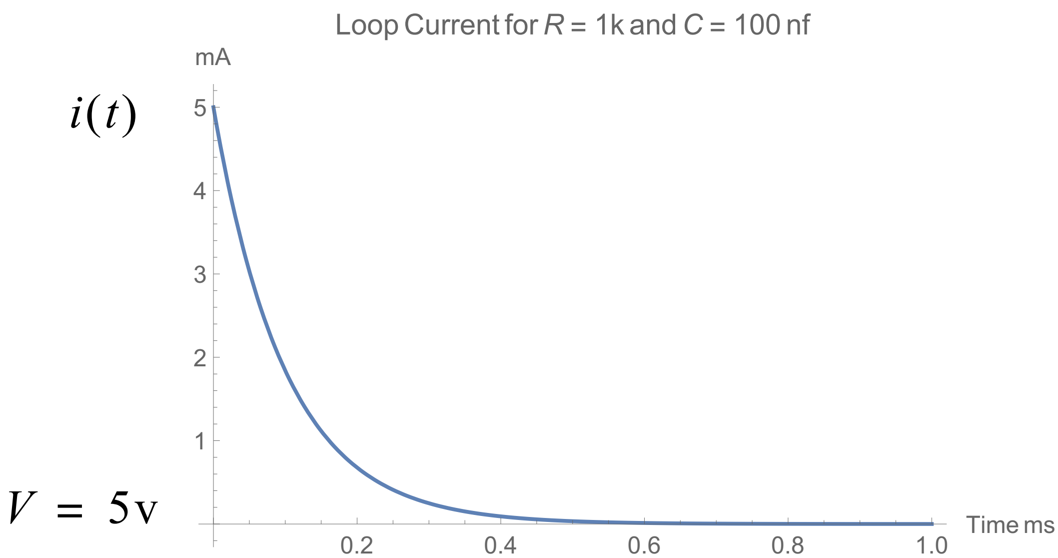
- Assume $v_c(0) = 0$
- At $t = \infty$ the capacitor can be removed and by inspection the current flow is zero, so no voltage drop across the resistor, meaning $v_c(\infty) = V$
- Here $R_{\text{eq}} = R$, so $\tau = RC$
- Finally,

$$\begin{aligned} v_c(t) &= V + [0 - V]e^{-t/\tau},\ t \geq 0 \\ &= V[1 - e^{-t/\tau}],\ t \geq 0 \end{aligned} \tag{6.9}$$



- Similarly, you can find the current $i_c(t)$ by noting that
  - $i_c(0) = V - v_c(0)/R = V/R$
  - $i_c(\infty) = 0$ since the capacitor is an open circuit
  - Finally,

$$i_c(t) = 0 + \left[\frac{V}{R} - 0\right]e^{-t/\tau} = \frac{V}{R}e^{-t/\tau},\ t \geq 0 \tag{6.10}$$

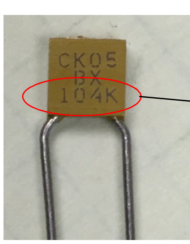


Capacitor Markings

- For fixed value capacitors the value is not always obvious
- For variable capacitors you generally have to consult the manufacturers data sheet
- Electronic Industries Association (EIA) Tips:
  - The value of the capacitor is denoted in picofarads for ceramic, film, and tantalum capacitors, but for aluminum electrolytics the value is denoted in micro-farads

MULTIPLIER USED ON EIA CAPACITOR MARKING CODE	
THIRD FIGURE	MULTIPLIER
0	1
1	10
2	100
3	1000
4	10 000
5	100 000
6	1 000 000

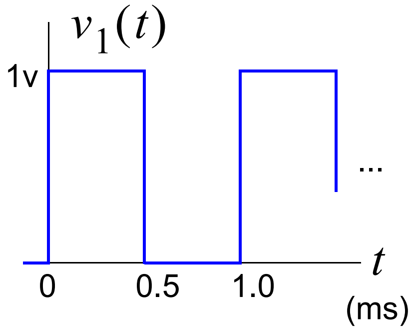
EIA TOLERANCE CAPACITOR MARKING CODE	
LETTER	TOLERANCE
Z	+80%, -20% - this is used with electrolytic capacitors where the minimum value is the major issue.
M	±20%
K	±10%
J	±5%
G	±2%
F	±1%
D	±0.5%
C	±0.25%
B	±0.1%

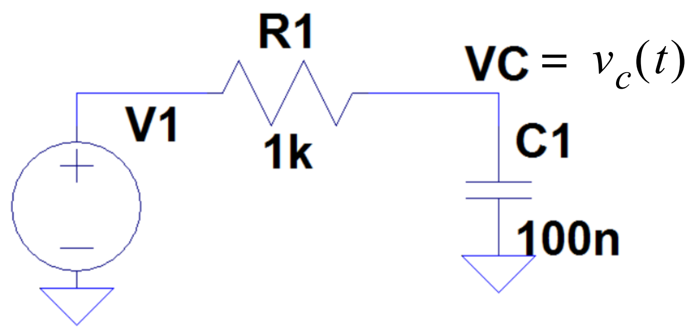


104K means this ceramic capacitor has value
$C = 10 \times 10^4$ pf $= 10^5$ pf $= 10^{-7}$ f = 100nf = 0.1uf.
The K means the tolerance is 10%.

Example 6.1: RC Lowpass Simulated and Measured

- In this example I consider LTspice simulation and then breadboard testing of the circuit shown below

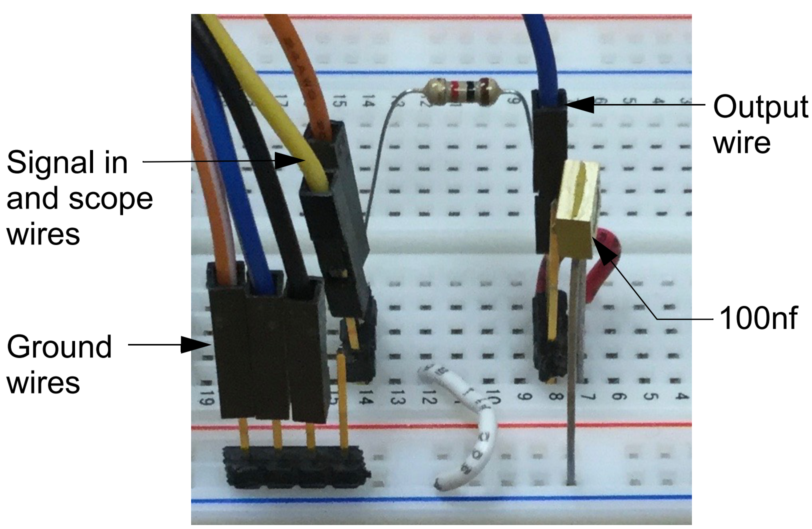




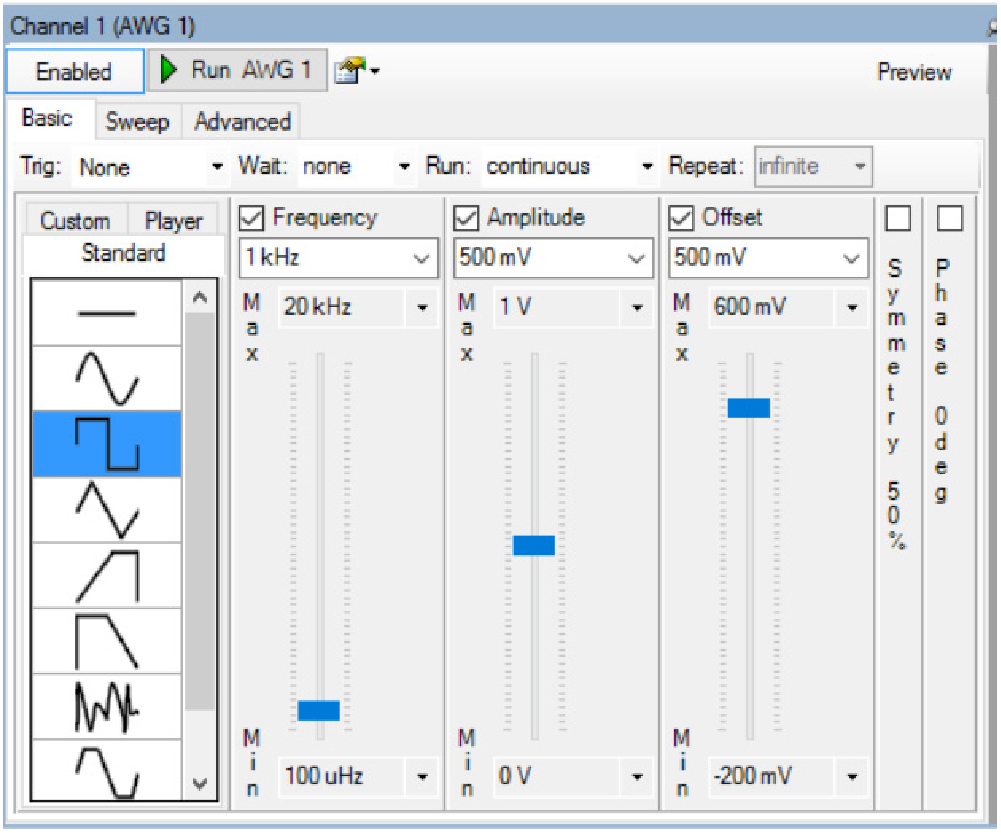
PULSE(0 1 0 100n 100n 0.5m 1m)

.tran 0 2m 0

- The objective is to find the voltage across the capacitor, $v_c(t)$ in response to a *rectangular pulse train* input having period of 1ms and amplitude swing of 0 to 1v
- In LTspice the input waveform is generated using the PULSE source
- The breadboard set-up is shown below:



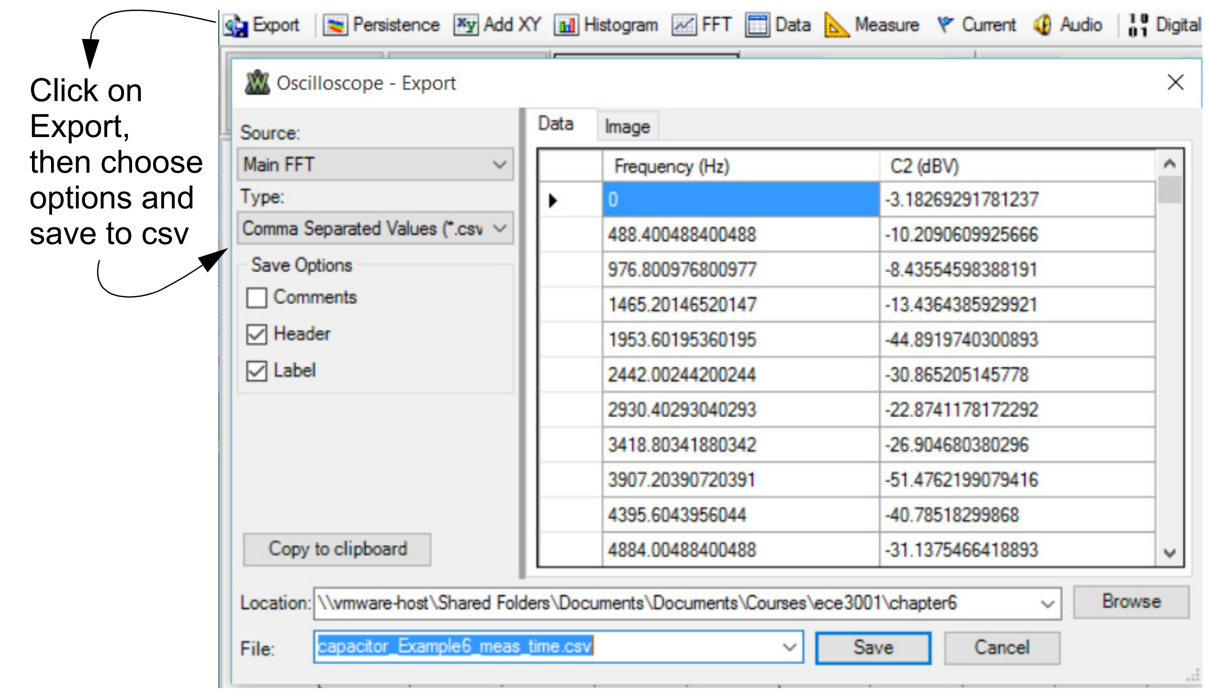
- For the Analog Discovery waveform generator a combination of a 0.5v peak amplitude and a 0.5v offset produces the desired waveform

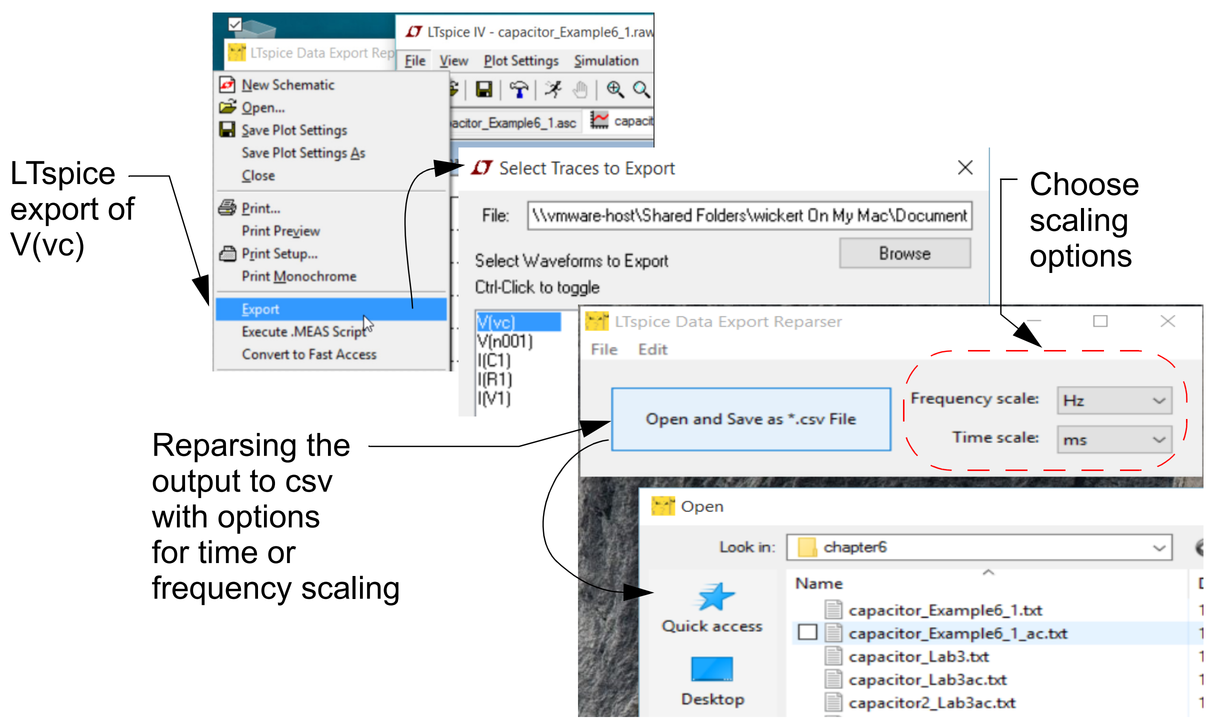


- The object now is to compare the simulated and measured $v_c(t)$ waveform on $0 \le t \le 2\,\text{ms}$
- The comparison is done using Excel to create an overlay plot after exporting raw data from LTspice and the Analog Discovery scope
  - The common file format chosen is *comma separated value* (csv)
  - The Analog Discovery supports this
  - LTspice does not provide this directly, but a small GUI

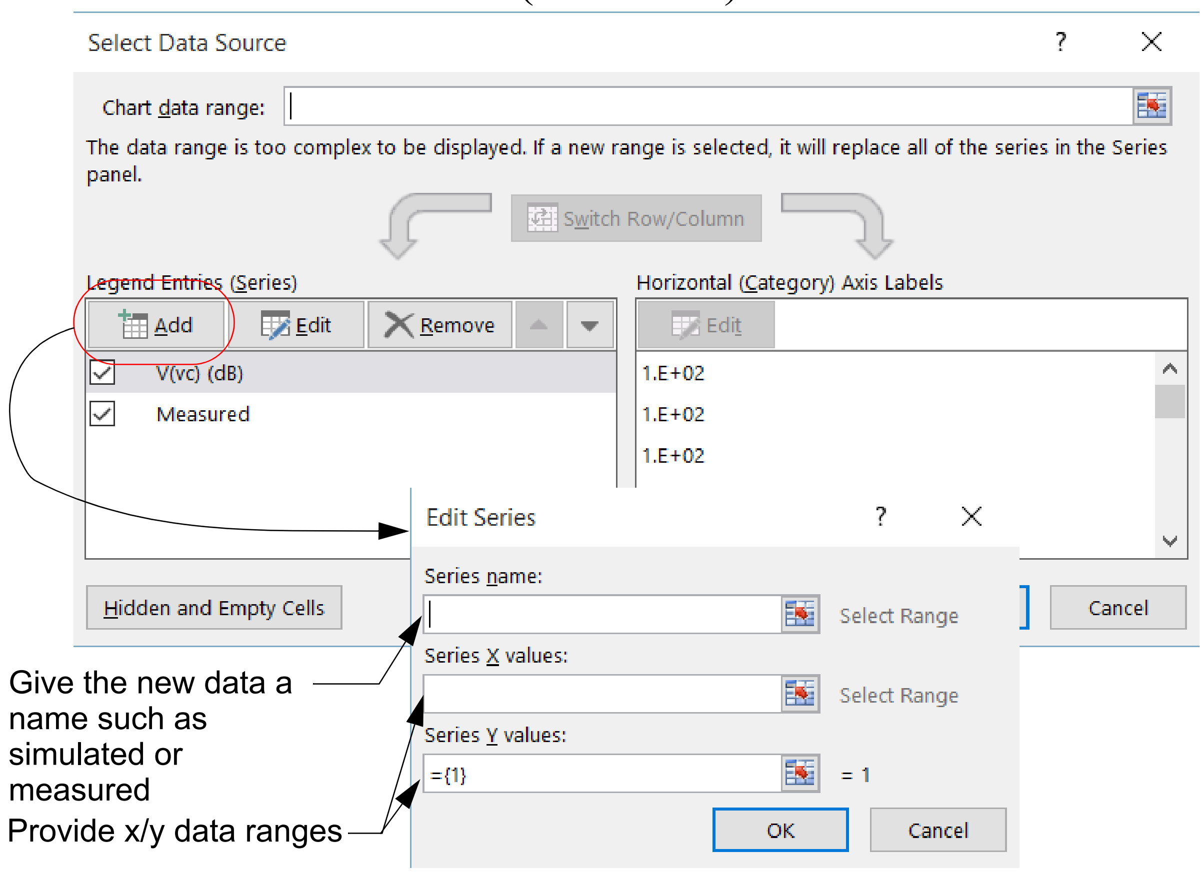
utility, `LTspice Data Export Reparser.exe`, is available to help convert the LTspice exported output to csv

- Exporting to data files:





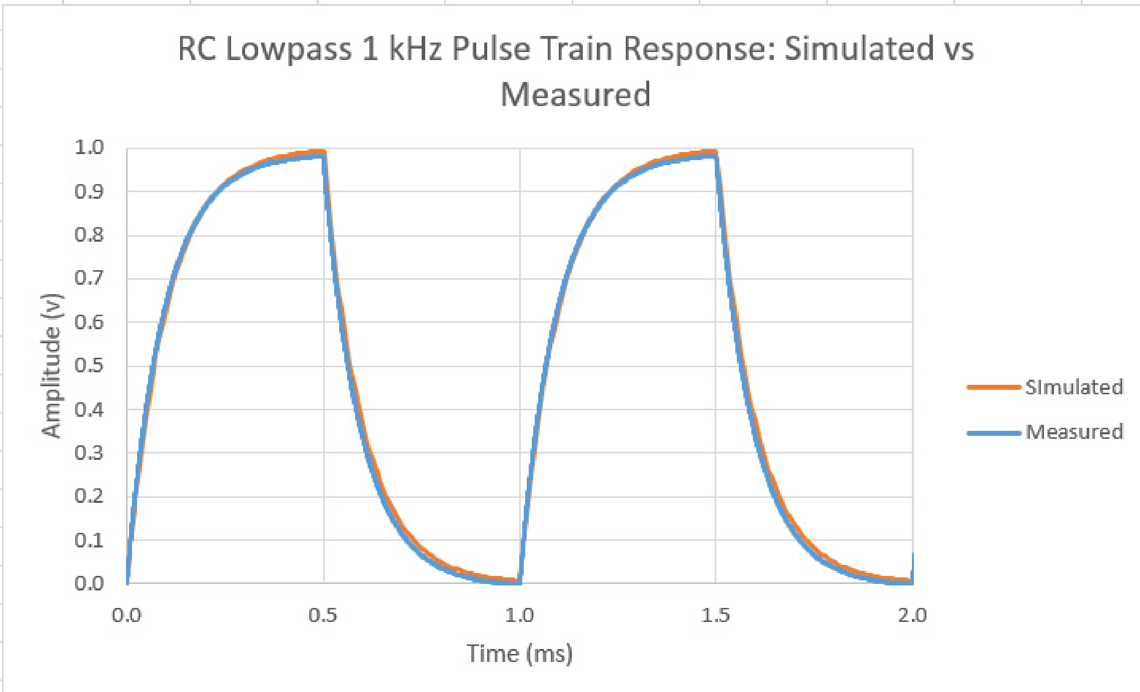
- Importing the csv files into Excel to create the overlay plot requires:
  - Import the data sets by choosing the **Data tab** and then data source From Text to ultimately place the data in an existing spreadsheet table
  - Highlight one of the two data sets and create a scatter plot
  - Right click over the plot and from the content menu choose **Select Data...** (see below)



  - Click on **Add** to add the second data set as x and y columns; click **OK** and you will then have an overlay plot
  - You will have to do some formatting make things look nice

- The results for the RC lowpass are shown below

	A	B	C	D	E	F
1	Time (s)	Time (ms)	C2 (V)		Time (ms)	V(vc)
2	1.10E-08	1.10E-05	0.000352708		0.00E+00	0.00E+00
3	2.61E-07	2.61E-04	0.002708978		1.43E-05	1.68E-05
4	5.11E-07	5.11E-04	0.006075078		2.86E-05	5.17E-05

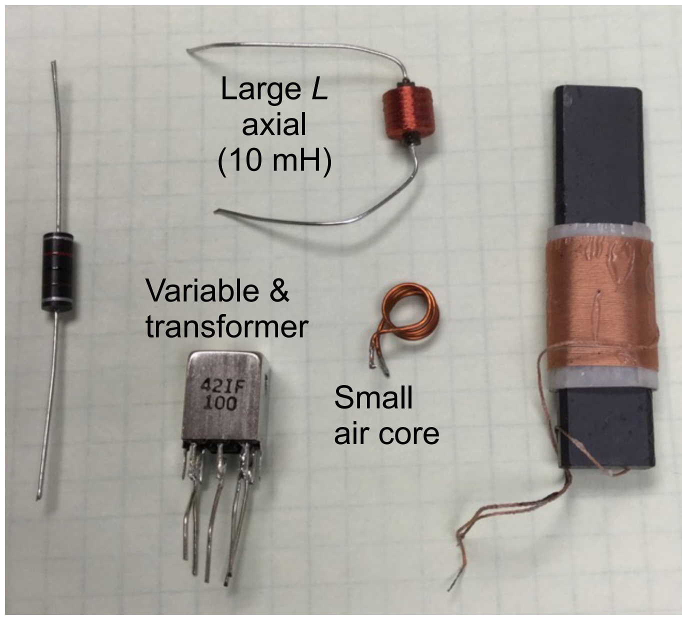


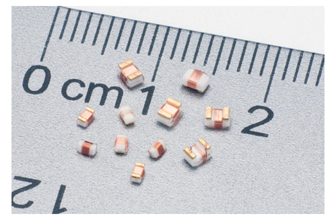
- The simulated and measured results compare favorably!
- The Excel file can be found on the Web Site under Chapter 6

Inductor

- A circuit element capable of storing energy in the magnetic field that is next to the conductors (coils) of the inductor

Axial with color code (1 mH)



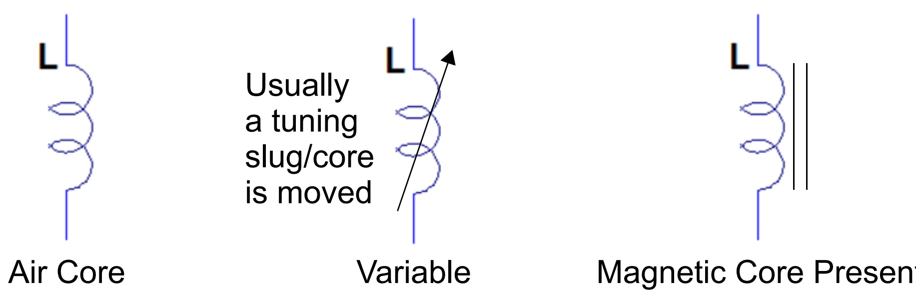


Various chip inductors

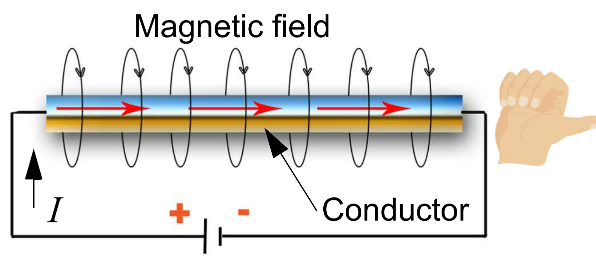
Loopstick AM antenna coil

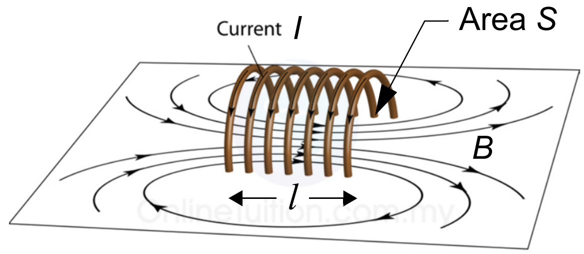
  - The inductance value, L, has units of Henries (or V s/A, weber/V, Wb/V)

- Schematic symbols:



- Basic principles[1]
  - Current flowing through a wire produces a magnetic field, B, circling the wire (use right-hand rule)
  - When the wire is tightly wound in a helix, flux linkages occur between the loops





- The *self inductance* (commonly inductance) is the ratio of magnetic flux linkages, Λ, over the current I [3]

$$L = \frac{\Lambda}{I}\text{H} \tag{6.11}$$

- For an N-turn solenoid structure having length l and cross section area S

$$L = \mu\frac{N^2}{l}S = \mu_0\mu_r\frac{N^2}{l}S\text{H} \tag{6.12}$$

  where μ_0 is the permeability of free space, $4\pi\times10^{-7}$ H/m, and μ_r is the relative permeability surrounding the coil

- The energy stored in an inductor is

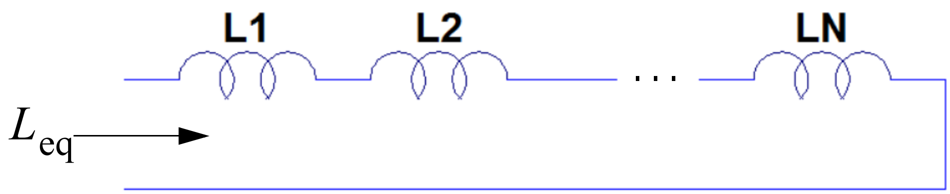
$$W = \frac{1}{2}LI^2\text{ J} \tag{6.13}$$

  with I being the current through the inductor

1. https://en.wikipedia.org/wiki/Inductor

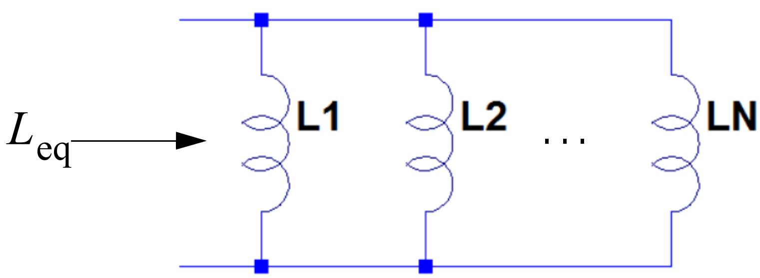
Parallel and Series Connections

- The formulas for equivalent inductance when inductors are placed in **series** is the same as for resistors



$$L_{eq} = L_1 + L_2 + \cdots + L_N \tag{6.14}$$

- The formulas for equivalent inductance when inductors are placed in **parallel** is the same as for resistors



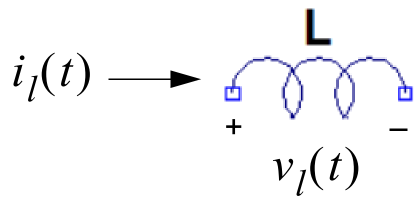
$$L_{eq} = L_1 \| L_2 \| \cdots \| L_N = \frac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \cdots + \dfrac{1}{L_N}} \tag{6.15}$$

- For the special case of just two inductors the equivalent value is just

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \tag{6.16}$$

Time Domain (Transient) Behavior

- The behavior of a inductor in a circuit is governed by the voltage across the inductor, the current through the inductor, and of course the inductance L



- The fundamental terminal relationship involves calculus:

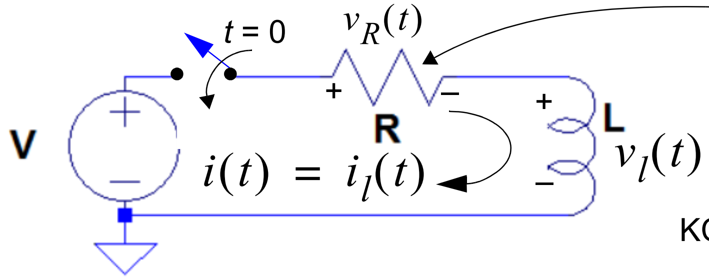
$$v_l(t) = L\frac{di_l(t)}{dt} \approx L\frac{i_l(t) - i_l(t - \Delta t)}{t - (t - \Delta t)} \tag{6.17}$$

or in words, the voltage across an inductor is L times the derivative of the current through the inductor (slope of the current through the inductor with respect to time)

- Using integration (from Calculus) you can solve for the current through the inductor

$$i_l(t) = \frac{1}{L}\int_{t_0}^{t} v_l(\alpha)d\alpha + i_l(t_0) \tag{6.18}$$

- When an inductor is placed in a circuit with resistors and other inductors (maybe capacitors too), you can use KCL and KVL (see notes Chapter 3) to solve for a voltage or current of interest



Note:

$$v_R(t) = Ri(t) = Ri_l(t)$$

$$v_l(t) = \frac{L}{R}\frac{dv_R(t)}{dt}$$

$$\text{KCL} \Rightarrow V = v_R(t) + \frac{L}{R}\frac{dv_R(t)}{dt}$$

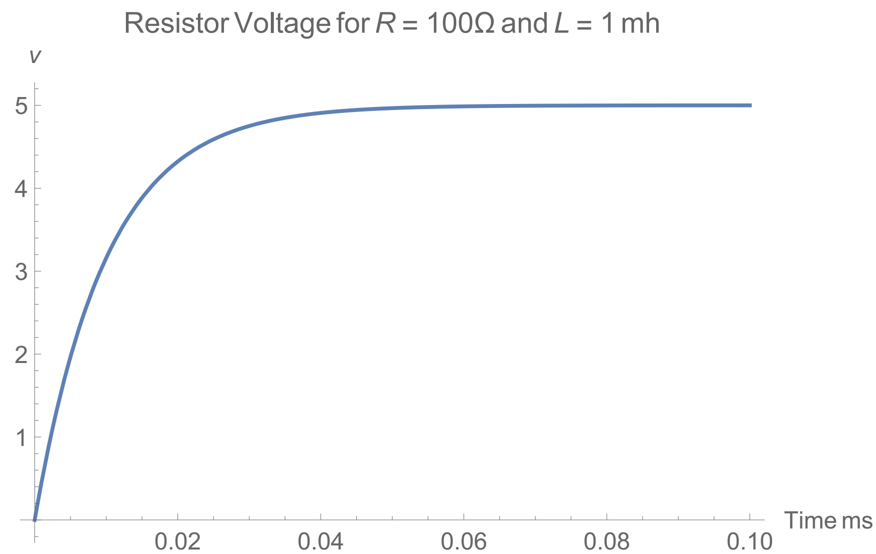
- Again you see in the RC circuit a first-order differential equation materializes
- As in the capacitor case, LTspice works well for circuit simulation and the inspection method (repeated (6.8)),

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}, t \geq 0 \tag{6.19}$$

  is very nice for simple first-order circuits
- Inductor behavior under the inspection method:
  - The current through an inductor cannot change instantly; it must charge up over time
  - At $t = 0$ the current through the inductor is $v_l(0)$
  - At $t = \infty$ you find $v_l(\infty)$ assuming the inductor is fully charged and the voltage drop is zero, making the inductor appear as a short circuit (inductor replaced by a short)
- The time constant $\tau = L/R_{\text{eq}}$, where R_{eq} is the equivalent resistance in series with the inductor
- Consider now the circuit above and find the voltage $v_R(t)$
  - Assume $i_l(0) = 0$
  - At $t = \infty$ the inductor can be shorted and by inspection the current flow is V/R, so $i_l(\infty) = V/R$ and $v_R(\infty) = V$
  - Here $R_{\text{eq}} = R$, so $\tau = L/R$
  - Finally,

$$v_R(t) = V + [0 - V]e^{-t/\tau},\ t \ge 0$$
$$= V[1 - e^{-t/\tau}],\ t \ge 0 \tag{6.20}$$

$v_R(t)$



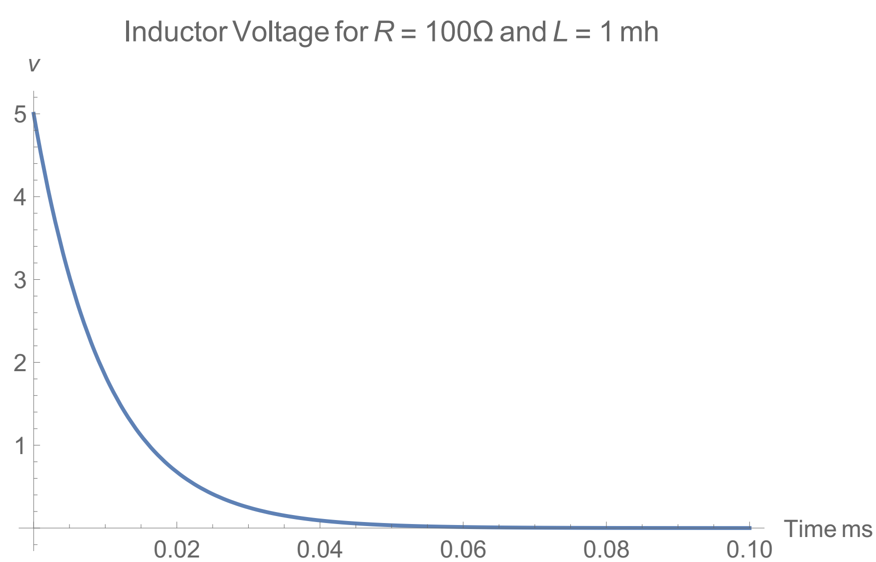
$V = 5\text{v}$

- You can find the inductor voltage $v_l(t)$ using KVL
  – Note: $V = v_R(t) + v_l(t) \Rightarrow v_l(t) = V - v_R(t)$

so

$$v_l(t) = V - V[1 - e^{-t/\tau}] = Ve^{-t/\tau},\ t \ge 0 \tag{6.21}$$

$v_l(t)$



$V = 5\text{v}$

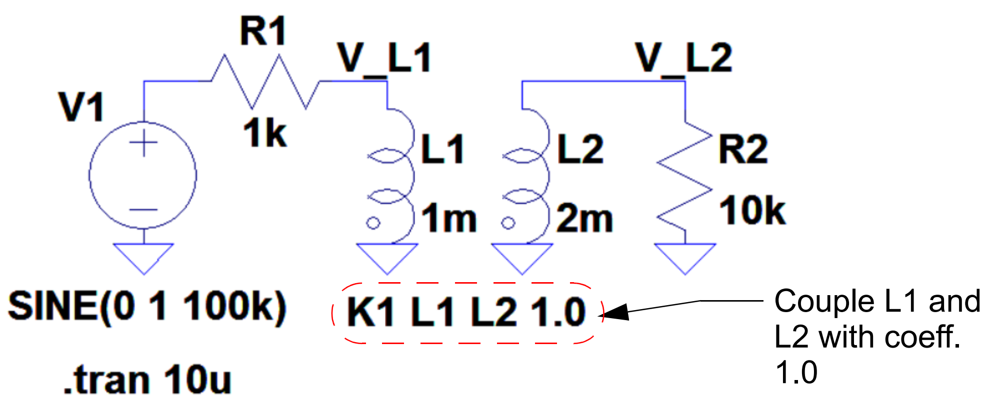
Mutual Inductance

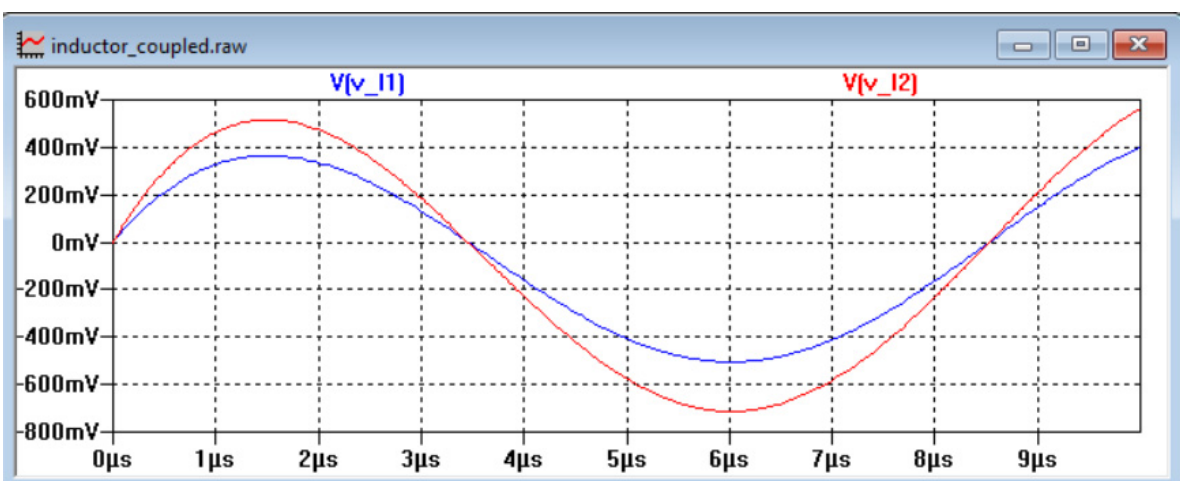
- When two coils are in close proximity on the same axis magnetic coupling occurs, the magnetic flux from one passes through the other
- *Mutual inductance* describes the inductance between the two coils as well as a *coupling coefficient*, ranging from 0 to 1 defines how tight the coupling is
  - To couple two inductors in LTspice the `K` directive is used

```
K L1 L2 K_value
```

Example 6.2: Quick Example

- Two inductors with a 1.0 coupling factor

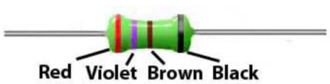




Inductor Markings

- For axial inductors a color similar to the resistor color code is often used
- For variable inductors you generally have to consult the manufacturers data sheet
- Axial Inductor Color Codes:

Standard 4-Band

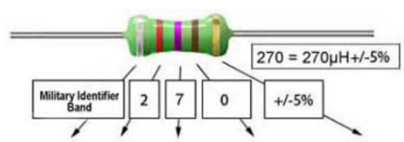


Therefore value = 27 x10 = 270µH +/-20%

Band	1	2	3	4
Meaning	1st Digit	2nd Digit	Multiplier (No. of zeros)	Tolerance %
Gold			x 0.1 (divide by 10)	+/-5%
Silver			x 0.01 (divide by 100)	+/-10%
Black	0	0	x1 (No Zeros)	+/-20%
Brown	1	1	x10 (0)	
Red	2	2	x100 (00)	
Orange	3	3	x1000 (000)	
Yellow	4	4	x10000 (0,000)	
Green	5	5		
Blue	6	6		
Violet	7	7		
Grey	8	8		
White	9	9		

Note: If no Band 4 is used, tolerance is also +/-20%

Military 5-Band



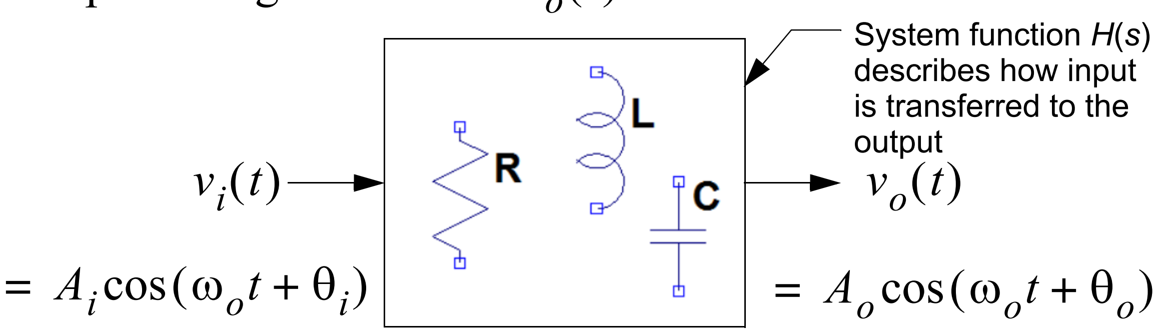
Band	1	2	3	4	5
Meaning (See Notes)	Mil. Spec.	Digit or Dec. point	Digit or Dec. point	Digit (or Multiplier)	Tolerance %
Gold		Decimal point	Decimal point		+/-5%
Silver	Always Silver double width				+/-10%
Black		0	0	0 (or x 1)	+/-20%
Brown		1	1	1 (or x 10)	+/-1%
Red		2	2	2 (or x 100)	+/-2%
Orange		3	3	3 (or x 1,000)	+/-3%
Yellow		4	4	4 (or x 10,000)	+/-4%
Green		5	5	5	
Blue		6	6	6	
Violet		7	7	7	
Grey		8	8	8	
White		9	9	9	

AC (alternating current) Steady State

- Moving beyond the transient analysis, radio circuit design needs to consider the steady-state behavior of circuits with R, L, and C elements
- The theme in this section is considering circuits where the input(s) are sinusoid signals that have been applied for a long time relative to the time it takes for transients to die out
- In notes Chapter 5 the use of the s-domain will be continued, so as to allow circuit modeling for steady-state input/output relationships, e.g., filters and amplifier coupling networks

Sinusoidal Steady-State

- To get a handle on steady-state circuit analysis, consider a generic R, L, C circuit with input voltage waveform v_i and output voltage waveform $v_o(t)$:



Linear Time Invariant Network/Circuit

- An s-domain relationship (assuming zero initial conditions) exists for this network which states that [2]

$$V_o(s) = V_i(s)H(s) \tag{6.22}$$

where $V_o(s)$ and $V_i(s)$ are s-domain node voltages in the circuit corresponding to $v_o(t)$ and $v_i(t)$ respectively, and $H(s)$ is the system function (or transfer function for $H(s = j\omega)$) relating the output to the input

- It could also be that currents or a mixture of voltage and current waveforms and their s-domain counter parts are being considered

• In any case, in steady state, the time domain response resulting from input $v_i(t)$, applied for a long time, produces output

$$v_o(t) = A_o \cos(\omega_o t + \theta_o) \tag{6.23}$$

where

$$\begin{aligned} A_o &= A_i |H(s{=}j\omega_o)| \\ \theta_o &= \theta_i + \angle H(s{=}j\omega_o) \end{aligned} \tag{6.24}$$

- **Note**: Since $H(s{=}j\omega_o)$ is a complex number, that means that just like a 2D vector, it has a length or magnitude and an angle to indicate its direction in the complex plane

• Drilling down on $H(s{=}j\omega)$ a bit more,

$$\begin{aligned} H(j\omega) &= H_{\text{Re}}(j\omega) + jH_{\text{Im}}(j\omega) \text{ (re/im)} \\ |H(j\omega)| &= \sqrt{H_{\text{Re}}^2(j\omega) + H_{\text{Im}}^2(j\omega)} \text{ (mag)} \\ \angle H(j\omega) &= \text{atan}\left[\frac{H_{\text{Im}}(j\omega)}{H_{\text{Re}}(j\omega)}\right] \text{ (phase/angle)} \\ H_{\text{Re}}(j\omega) &= |H(j\omega)| \cos \angle H(j\omega) \text{ (re)} \\ H_{\text{Im}}(j\omega) &= |H(j\omega)| \sin \angle H(j\omega) \text{ (im)} \end{aligned} \tag{6.25}$$

- **Conclusion**: Under sinusoidal steady state conditions the analysis reduces down to solving circuit equations to find $|H(j\omega)|$ and $\angle H(j\omega)$ as a function of frequency $\omega = 2\pi f$, where f is in Hz
- **First Step**: Find $H(s) = V_o(s)/V_i(s)$ for a given circuit input and output connection point
- **Second Step**: Set $s = j\omega = j2\pi f$ and solve for the magnitude $|H(j\omega)|$ and phase $\angle H(j\omega)$
  - The last step requires some complex arithmetic, but in the examples that follow, I will show you some tricks to keep the math easier by having Excel do the heavy lifting

---

Example 6.3: Quick Example

- Suppose that

$$H(s) = \frac{1}{1 + s \cdot 10^{-4}} \tag{6.26}$$

so

$$H(j\omega) = H(j2\pi f) = \frac{1}{1 + j2\pi f \cdot 10^{-4}} \tag{6.27}$$

- Next suppose that $v_i(t) = 2\cos(2\pi f_o t + 45°)$ and consider f_o taking on values of 100 Hz and 15 kHz
- To find $v_o(t)$ you need to use (6.24) with $H(j2\pi f)$ evaluated at 100 Hz and 15 kHz

- To help with the complex arithmetic you can use Excel (yes Python with the Scipy stack, Mathematica, and MATLAB all work very well)

Table 6.1: Complex arithmetic in Excel (a partial list)

Function	Description	Example
COMPLEX(x,y,"j")	Returns/creates a complex number z = x + jy	Given: z = x +jy = 3 + 4j COMPLEX(3,4,"j") returns 3+4j
IMPRODUCT(z1,z2)	Returns the complex number z1 x z2	Given: z1=1+2j, z2=3-4j IMPRODUCT(z1,z2) = 11+2j IMPRODUCT(z1,3) = 3+6j
IMSUM(z1,z2)	Returns the complex number z1 + z2	Given: z1=1+2j, z2=3-4j IMSUM(z1,z2) = 4-2j IMSUM(1,z2) = 4-4j
IMDIV(z1,z2)	Returns the complex number z1/z2	Given: z1=1+2j, z2=3-4j IMDIV(z1,z2) = -0.2+0.4j IMDIV(1,z2) = 0.12+0.16j
IMABS(z1)	Returns the magnitude (a real number) |z1|	Given: z2=3-4j IMABS(z2) = 5 i.e., ($\sqrt{3^2+4^2}$)
IMARGUMENT(z1)	Returns the angle in radians (a real number) angle(z1)=arg(z1)	Given: z2=3-4j IMARGUMENT(z2) = -0.9273 which is the angle of z2 in radians
IMPRODUCT (Z_mag,COMPLEX (COS(Z_ang*PI()/ 180),SIN(Z_ang)*PI( )/180,"j"))	Convert a complex number given as magnitude and angle (deg) to real/imaginary form	Given z4 = 5/_50 (5 angle 50 deg) IMPRODUCT(5,COMPLEX(COS(50*PI()/ 180),SIN(50*PI()/180))) = 3.214 + 3.830j
IMREAL(z1)	Returns the real part of a complex number	Given z1 = 3 - 4j IMREAL(z1) = 3
IMIMAGINARY(z1)	Returns the imaginary part of a complex number	Given z1 = 3 - 4j IMIMAGINARY(z1) = -4
IMPOWER(z1,N)	Returns a complex number raised to an integer power $z1^N$	Given: z1=1+2j IMPOWER(z1,3) = -11-2j which is $z1^3$

- In this example you use Excel complex arithmetic to directly evaluate

$$H(s = j2\pi f) = \left.\frac{1}{1 + s \cdot 10^{-4}}\right|_{s = j2\pi f} \tag{6.28}$$

in a sequence of steps

- Since s is complex you must use the `IMxx` functions!
- The calculation steps are shown below:

C2 | =IMDIV(1,IMSUM(1,IMPRODUCT(B2,0.0001)))

	A	B	C	D	E	F
1	f	s=j*2*pi*f	H(s)	|H(s)|	|H(s)|dB	Angle[H(s)]
2	100	628.318530717959j	0.996067682407173-0.0625847782705717j	0.99803	-0.0171	-3.5953
3	15000	94247.7796076938j	0.0111325797208916-0.104922091999967j	0.10551	-19.5340	-83.9434

Step 1: *Form s in col B==>* =COMPLEX(0,2*PI()*A2,"j")

Step 2: *Form H(s) in col C==>* =IMDIV(1,IMSUM(1,IMPRODUCT(B2,0.0001)))

Step 3: *Form |H(s)| in col D==>* =IMABS(C2)

$$\frac{1}{1 + s \cdot 0.0001}$$

Step 4: *Form |H(s)| in dB in col E==>* =20*LOG10(D2)

Step 5: *Form the angle of H(s) in deg in col F==>* =IMARGUMENT(C2)*180/PI()

- As a check, convert the magnitude and phase back to real and imaginary parts

=IMREAL(IMPRODUCT(D3,COMPLEX(COS(F3*PI()/180),SIN(F3*PI()/180),"j")))

C	D	E	F	G	H
d Phase at 100 Hz and 15kHz					
)	|H(s)|	|H(s)|dB	Angle[H(s)]	Re{H(s)}	Im{H(s)}
96067682407173-0.0625847782705717j	0.99803	-0.0171	-3.5953	0.996068	-0.06258
11325797208916-0.104922091999967j	0.10551	-19.5340	-83.9434	0.011133	-0.10492

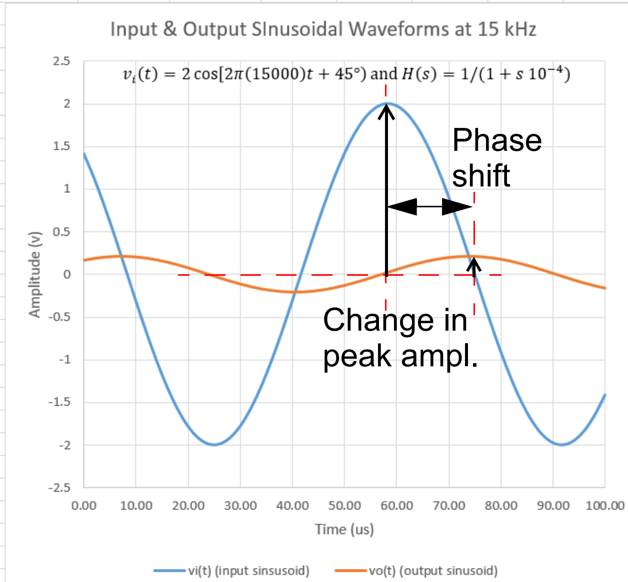
=IMREAL(IMPRODUCT(D3,COMPLEX(COS(F3*PI()/180),SIN(F3*PI()/180),"j")))
=IMIMAGINARY(IMPRODUCT(D3,COMPLEX(COS(F3*PI()/180),SIN(F3*PI()/180),"j")))

- From the spreadsheet answers in columns D and F, you can now write expressions for the output waveform

$$
\begin{aligned}
v_o^{100\text{Hz}}(t) &= 2 \cdot 0.99803 \cos[2\pi(100)t - 3.60° + 45°] \\
v_o^{15\text{kHz}}(t) &= 2 \cdot 0.10551 \cos[2\pi(15000)t - 83.95° + 45°]
\end{aligned}
\tag{6.29}
$$

- The 15kHz input/output waveforms are plotted below:

6	Plotting the Time Domain Waveforms Similar to Ltspice Tran		
7	t (us)	vi(t) (input sinsusoid)	vo(t) (output sinusoid)
8	0.00	1.41E+00	1.64E-01
9	2.00	1.12E+00	1.86E-01
10	4.00	7.94E-01	2.01E-01
11	6.00	4.36E-01	2.10E-01
12	8.00	6.28E-02	2.10E-01
13	10.00	-3.13E-01	2.04E-01
14	12.00	-6.77E-01	1.90E-01
15	14.00	-1.02E+00	1.69E-01
16	16.00	-1.32E+00	1.43E-01
17	18.00	-1.58E+00	1.11E-01
18	20.00	-1.78E+00	7.54E-02
19	22.00	-1.92E+00	3.72E-02
20	24.00	-1.99E+00	-2.42E-03
21	26.00	-1.99E+00	-4.19E-02
22	28.00	-1.92E+00	-7.99E-02
23	30.00	-1.78E+00	-1.15E-01
24	32.00	-1.58E+00	-1.46E-01
25	34.00	-1.32E+00	-1.72E-01
26	36.00	-1.02E+00	-1.92E-01
27	38.00	-6.77E-01	-2.05E-01
28	40.00	-3.13E-01	-2.11E-01
29	42.00	6.28E-02	-2.09E-01
30	44.00	4.36E-01	-2.00E-01
31	46.00	7.94E-01	-1.84E-01
32	48.00	1.12E+00	-1.61E-01



- See the file `chapter6_freq_resp.xlsx` for more details

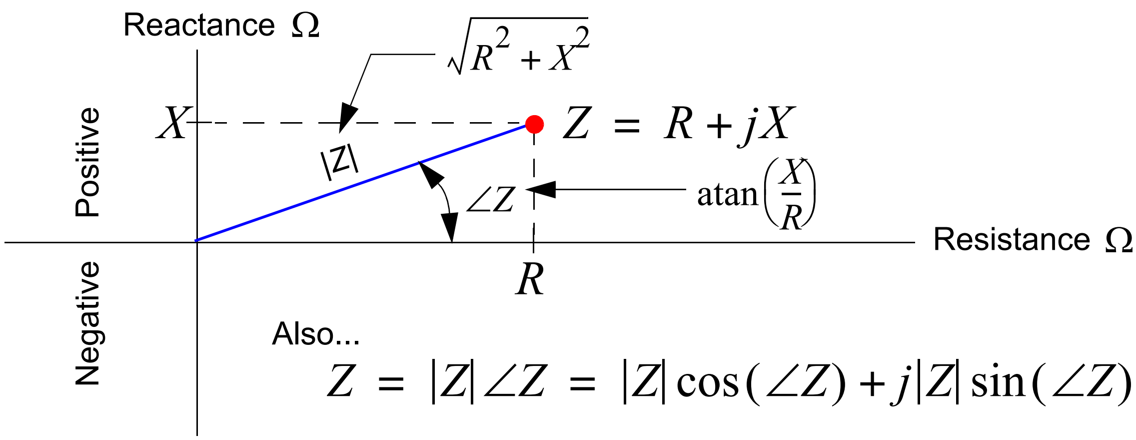
Impedance

- Move from resistor only circuits to *R*, *L*, and *C* circuits
- The concept of impedance for *R*, *L*, and *C* networks allows you work out *Step 1* and then move on to complex arithmetic tasks associated with step 2
- Under the AC steady state assumptions described at the beginning of this section, the concept of resistance as devel-

oped for DC circuits, morphs into impedance, a complex number (recall $j = \sqrt{-1}$)

$$Z = R + jX \tag{6.30}$$

which can be viewed in the plane as a 2D vector having resistance, R, on the real axis (x-axis) and reactance, X, on the imaginary axis (y-axis)



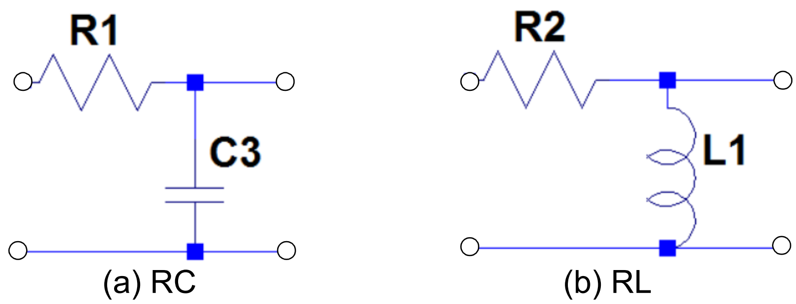
- From a more detailed study of the terminal relationships for capacitors and inductors [2], it can be shown that the following relationships hold:

Table 6.2: Reactance, impedance, and *s*-domain impedance

Element	Resistance/ Reactance	Impedance	*s*-Plane Impedance
Resistor	R ohms	R	R
Inductor (reactive)	$\omega L = 2\pi f L$ ohms	$j\omega L$	sL
Capacitor (reactive)	$\frac{-1}{\omega C} = \frac{-1}{2\pi f}$ ohms	$\frac{1}{j\omega C} = -\frac{j}{\omega C}$	$\frac{1}{sC}$

- Impedance has units of ohms, both resistance and reactance
- As shown in the figure above, you can also view impedance as a magnitude (vector length) and angle relative the real or resistance axis
- **Good news**, Ohms Law still works!
- LTSpice handles all of this flawlessly
- **R, L, C and Simple Insights**: The magnitude of Z
  - Both R and $|R|$ remain constant with frequency
  - An inductor is purely reactive having $|z| = |X_L| = 2\pi f L$, which is *proportional* to frequency
  - At DC the inductor is like a short circuit, while at high frequencies the inductor has very high impedance, ultimately infinite
  - A capacitor is also purely reactive having $|Z| = |-1/(2\pi f C)| = 1/(2\pi f C)$, which is *inversely proportional* to frequency
  - At DC the capacitor has infinite reactance making it act like an open circuit of DC block, while at high frequencies the reactance become very small, ultimately zero like a short circuit

- **Test Your Intuition**:



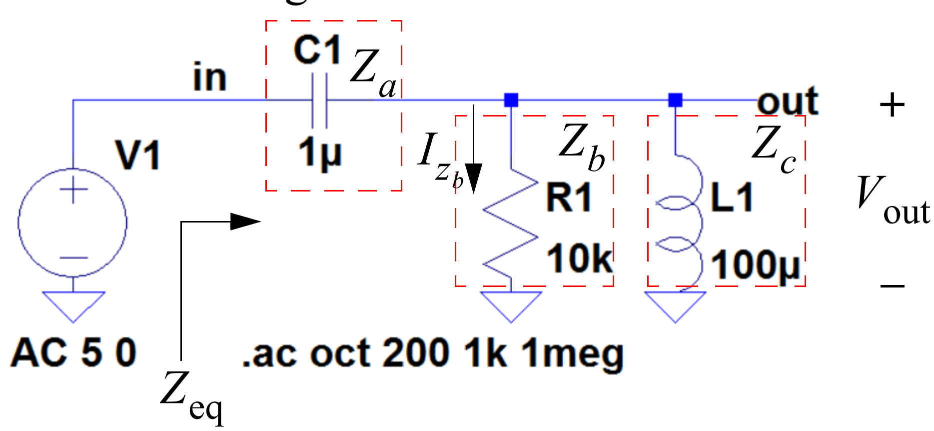
  – Compare high frequency and low frequency behavior of $|H(j2\pi f)|$ of the *RC* and *RL* circuits

  – **RC**: Low frequencies pass through as C_3 appears as an open, while at high frequencies the reactance of C_3 is small making the voltage divider action heavily attenuate the input signal; **overall a lowpass filter**

  – **RL**: Low frequencies are heavily attenuated here as the inductive reactance is very small at low frequencies, while at high frequencies the reactance of L_1 is large meaning that he voltage divider ratio magnitude is close to one; **overall a highpass filter**

Example 6.4: A Collection of *R*, *L*, and *C* Elements

- Consider the following circuit:



- Start by finding the impedance looking into the circuit
- No problem, if you consider the elements as having impedances Z_a, Z_b, and Z_c, and then use the circuit reduction technique developed for resistors (now for impedances):

$$Z_{\text{eq}} = Z_a + Z_b \| Z_c = Z_a + \frac{Z_b Z_c}{Z_b + Z_c} \tag{6.31}$$

- Next replace the Z values with the component specific s-domain impedance

$$Z_{\text{eq}}(s) = \underbrace{\frac{1}{sC_1}}_{Z_a} + \underbrace{\frac{R_1 \cdot sL_1}{R_1 + sL_1}}_{Z_{bc}} \tag{6.32}$$

- Now find $Z_{\text{eq}}(j2\pi f) = R_{\text{eq}} + jX_{\text{eq}}$ for $f = 1\,\text{kHz}$ and 1 MHz
- Complex arithmetic is required here so use Excel, or Python, etc.
  – An Excel solution is shown below

=IMSUM(F2,G2) ⟵ $Z_{\text{eq}} = Z_{\text{a}} + Z_{\text{bc}}$

A	B	C	D	E	F	G	H	I
C1	R1	L1	f	s	Za	Zab	Zeq	
1.00E-06	1.00E+04	1.00E-04	1.00E+03	6283.18530717959j	-159.154943091895j	0.0000394784174485(	0.0000394784174485(	29-15
			1.00E+06	6283185.30717959j	-0.159154943091895j	39.3231759282749+62	39.3231759282749+625.6886	

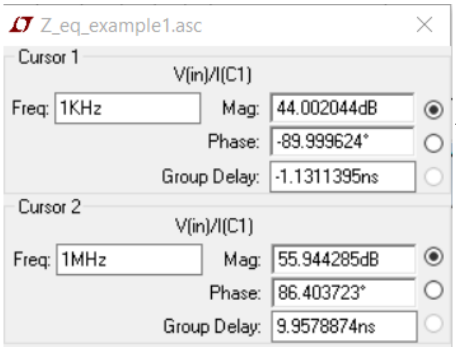
Zeq(1kHz) = 0.0000394784174485029-158.526624563658j

Zeq(1MHz) = 39.3231759282749+625.688627762625j

Note: F2 =IMDIV(1,IMPRODUCT(E2,A2))

G2 = IMDIV(IMPRODUCT(B2,IMPRODUCT(E2,C2)), IMSUM(B2,IMPRODUCT(E2,C

– Compare with a direct calculation in LTspice



Magnitude and Phase Form from LTspice is then converted to real/imaginary or resistance/reactance in ohms

From Ltspice		
9.70864722969463E-15-158.489319246112j		
39.3126653698025+625.379444610327j		

cell equations

Convert from dB

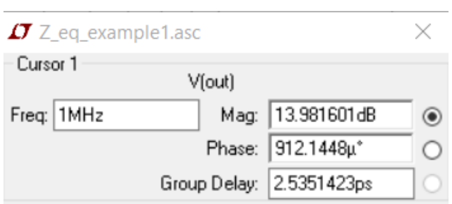
=IMPRODUCT(POWER(10,44/20),COMPLEX(COS(-90*PI()/180),SIN(-90*PI()/180),"j"))

=IMPRODUCT(POWER(10,55.94/20),COMPLEX(COS(86.403*PI()/180),SIN(86.403*PI()/180)))

- The two results agree!
- Next calculate the output voltage using the voltage divider relationship at $f = 1\,\text{MHz}$

$$V_{\text{out}}(s) = V \cdot \frac{Z_{bc}}{Z_a + Z_{bc}} \tag{6.33}$$

– Calculate using Excel and check with LTspice



Vout (theory)	
Mag	Angle(deg)
5.001266835	0.000912122
Vout(Ltspice)	
5.00034535	0

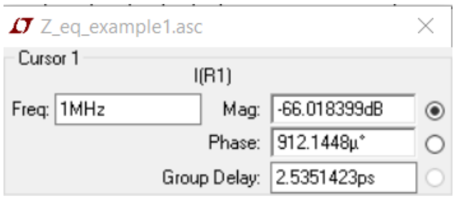
=5*IMABS(IMDIV(G3,IMSUM(F3,G3)))

=IMARGUMENT(IMDIV(G3,IMSUM(F3,G3)))*180/PI()

- Finally, calculate the current through the resistor or impedance Z_b at $f = 1\,\text{MHz}$

$$I_{R_1} = \frac{V_{\text{out}}}{R_1} = \frac{V}{R_1} \cdot \frac{Z_{bc}}{Z_a + Z_{bc}} \tag{6.34}$$

– Calculate using Excel and check with LTspice



IR2(theory)	
Mag	Angle(deg)
0.000500127	0.000912122
IR2(LTspice)	
0.000500035	9.12E-04

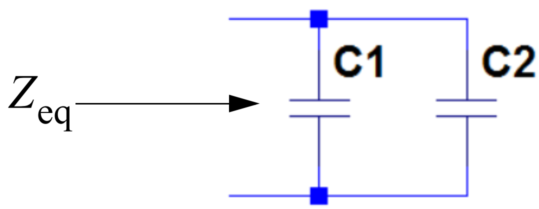
=5/1e4*IMABS(IMDIV(G3,IMSUM(F3,G3)))

=IMARGUMENT(IMDIV(G3,IMSUM(F3,G3)))*180/PI()

- The theory and simulated results are in agreement!

Example 6.5: Use Impedance to Show Capacitors add in Parallel

- Earlier it was stated that the total capacitance of capacitors connected in a parallel add



$$C_{\text{eq}} \stackrel{?}{=} C_1 + C_2$$

- Start by finding the impedance Z_{eq} looking into the parallel connection

$$Z_{\text{eq}}(s) = Z_{C_1} \| Z_{C_2} = \frac{\dfrac{1}{sC_1} \cdot \dfrac{1}{sC_2}}{\dfrac{1}{sC_1} + \dfrac{1}{sC_2}} \tag{6.35}$$

$$= \frac{1}{sC_1 + sC_2} = \frac{1}{s(C_1 + C_2)} = \frac{1}{sC_{\text{eq}}}$$

where $C_{\text{eq}} = C_1 + C_2$

- Proof complete!

- **Note**: You can use this same approach to show the series formula for capacitors and the series and parallel formulas for inductors

Frequency Response

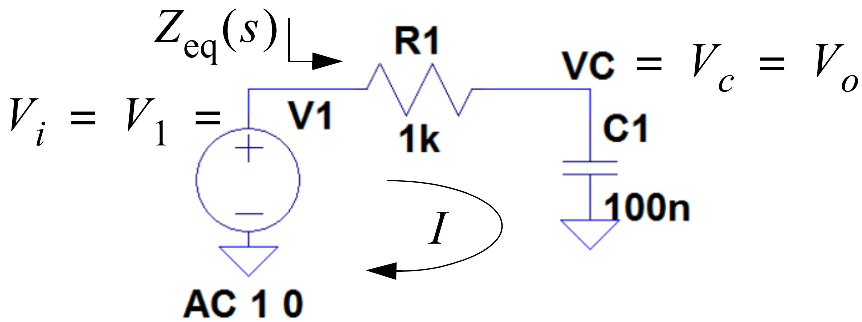
- The frequency response follows by taking the *transfer function*, which relates the input to the output via some $H(s)$, and replacing s with $j\omega = j2\pi f$
  - **Note**: You can also consider $Z(s)$, $V(s)$, or $I(s)$ under the same substitution, but the term transfer function no longer applies, you simply have the given quantity, impedance, voltage, or current, as a function of frequency f
- In Example 6.4 this applies to the calculation of the output voltage in terms of the input voltage, that is

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{Z_{bc}(s)}{Z_a(s) + Z_{\text{bc}}(s)} \tag{6.36}$$

  - Upon substitution you have the transfer function from input to output as a function of frequency
- In radio receiver electronics found in this class, the transfer function is generally a filter or amplifier in the signal processing chain that moves the received signal captured by the antenna to the speaker/headphone jack

Example 6.6: RC Network (Lowpass Filter) Revisited

- Consider the following circuit:



  – **Note**: The model above has $V_i = 1\angle 0°$, so effectively

$$v_i(t) = 1 \cdot \cos(\omega t + 0°) \tag{6.37}$$

- To solve for V_o you can use the voltage divider formula, now using impedances
- The output voltage (voltage across the capacitor) is found from the voltage divider relationship for impedances:

$$V_o = V_i \cdot \frac{Z_C}{Z_R + Z_C} = V_i \cdot \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = V_i \cdot \frac{1}{1 + sRC} \tag{6.38}$$

  – Too easy?

- To find V_0 as a function of the input sinusoid frequency let $s = j2\pi f$

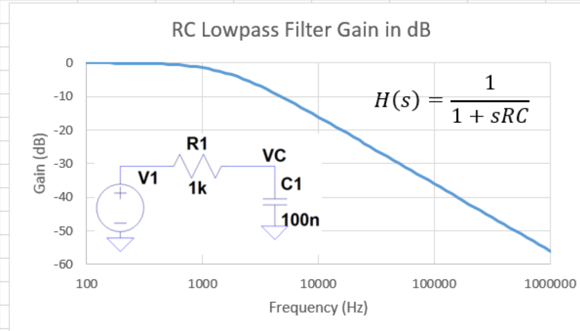
$$V_0 = V_i \cdot \frac{1}{1 + j2\pi fRC} \tag{6.39}$$

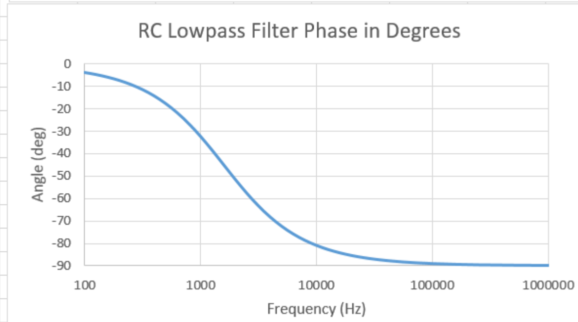
- The objective is to plot the magnitude and phase (angle) of V_o versus f

- Using the complex functions in Excel you can plot magnitude and phase as follows

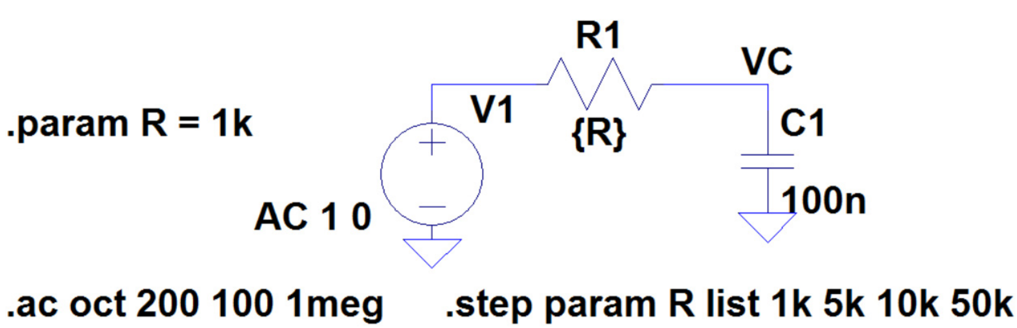
D2 =IMDIV(1,IMSUM(1,IMPRODUCT(C2,G2)))

	A	B	C	D	E	F	G
1	x	f = 10^x	s = j*2*PI*f	H(j2pi f)	HdB(j2pi f)	Phase H(f)	RC
2	2	100	628.318530717959j	0.9960676	-0.01711	-3.59527	0.0001
3	2.05	112.2018	704.984986645445j	0.9950545	-0.02153	-4.03259	
4	2.1	125.8925	791.006165022012j	0.9937819	-0.02709	-4.52271	
5	2.15	141.2538	887.523514621322j	0.9921845	-0.03408	-5.07185	
6	2.2	158.4893	995.817762032063j	0.9901808	-0.04285	-5.68687	
7	2.25	177.8279	1117.32590612166j	0.9876697	-0.05388	-6.37536	
8	2.3	199.5262	1253.66028613816j	0.9845265	-0.06773	-7.14566	
9	2.35	223.8721	1406.6299764725j	0.9805978	-0.08509	-8.00686	
10	2.4	251.1886	1578.26479197648j	0.9756961	-0.10685	-8.96881	
11	2.45	281.8383	1770.84222237266j	0.9695946	-0.1341	-10.0421	
12	2.5	316.2278	1986.91765315922j	0.9620209	-0.16815	-11.2378	
13	2.55	354.8134	2229.35827402299j	0.9526527	-0.21065	-12.5678	
14	2.6	398.1072	2501.38112470457j	0.9411152	-0.26357	-14.0437	
15	2.65	446.6836	2806.59578316113j	0.9269818	-0.32929	-15.6773	
16	2.7	501.1872	3149.05226247286j	0.9097812	-0.41063	-17.4795	
17	2.75	562.3413	3533.2947520559j	0.8890139	-0.51091	-19.4598	
18	2.8	630.9573	3964.421916295j	0.8641799	-0.63396	-21.6255	
19	2.85	707.9458	4448.15455072215j	0.8348215	-0.78406	-23.9802	
20	2.9	794.3282	4990.91149349751j	0.8005815	-0.96594	-26.5234	
21	2.95	891.2509	5599.89479949198j	0.7612735	-1.18459	-29.2484	
22	3	1000	6283.18530717959j	0.7169568	-1.44507	-32.1419	
23	3.05	1122.018	7049.84986645445j	0.6680009	-1.75223	-35.1833	
24	3.1	1258.925	7910.06165022013j	0.6151231	-2.11038	-38.3442	
25	3.15	1412.538	8875.23514621322j	0.5593786	-2.52294	-41.5898	
26	3.2	1584.893	9958.17762032063j	0.5020954	-2.99214	-44.8799	
27	3.25	1778.279	11173.2590612166j	0.4447573	-3.51877	-48.1716	
28	3.3	1995.262	12536.6028613816j	0.3888532	-4.10214	-51.4219	
29	3.35	2238.721	14066.299764725j	0.3357273	-4.74013	-54.5903	





- The RC lowpass filter can easily be analyzed in LTspice and experimentally measured using the Analog Discovery
- The complete LTspice AC analysis schematic is shown below:

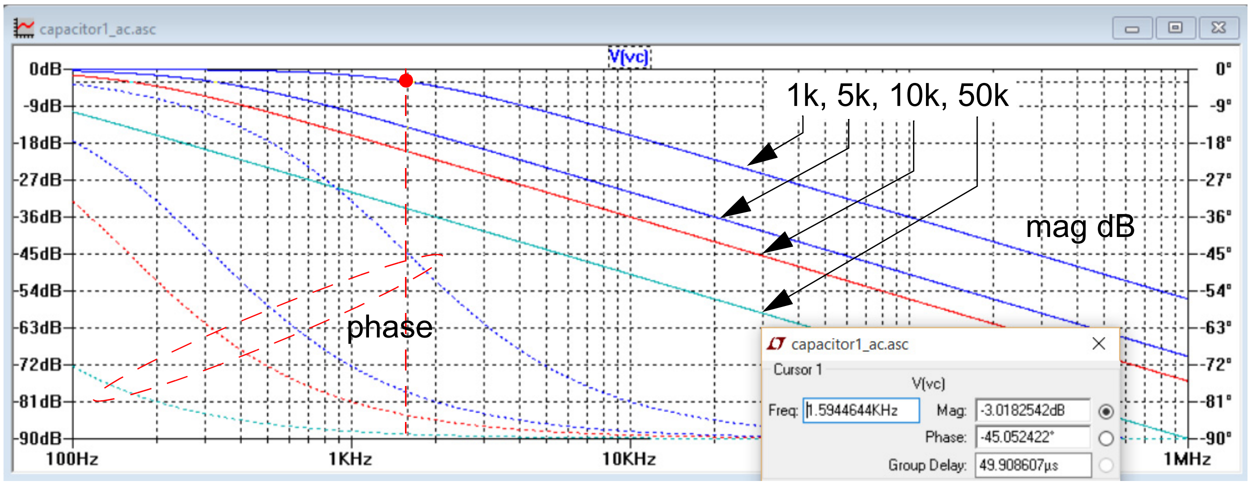


  – Here I have also included the use of `.step` to vary the value of R_1 using the `list` option

– A second way of using `.step` is

```
.step param R Rstart Rstop Rstepsize
```

– The LTspice plot showing the RC circuit frequency response is shown below (cursor at the -3dB point)



– **Note**: The frequency at which the magnitude response is *down* 3dB is $1/(2\pi RC) = 1.591\,\text{kHz}$

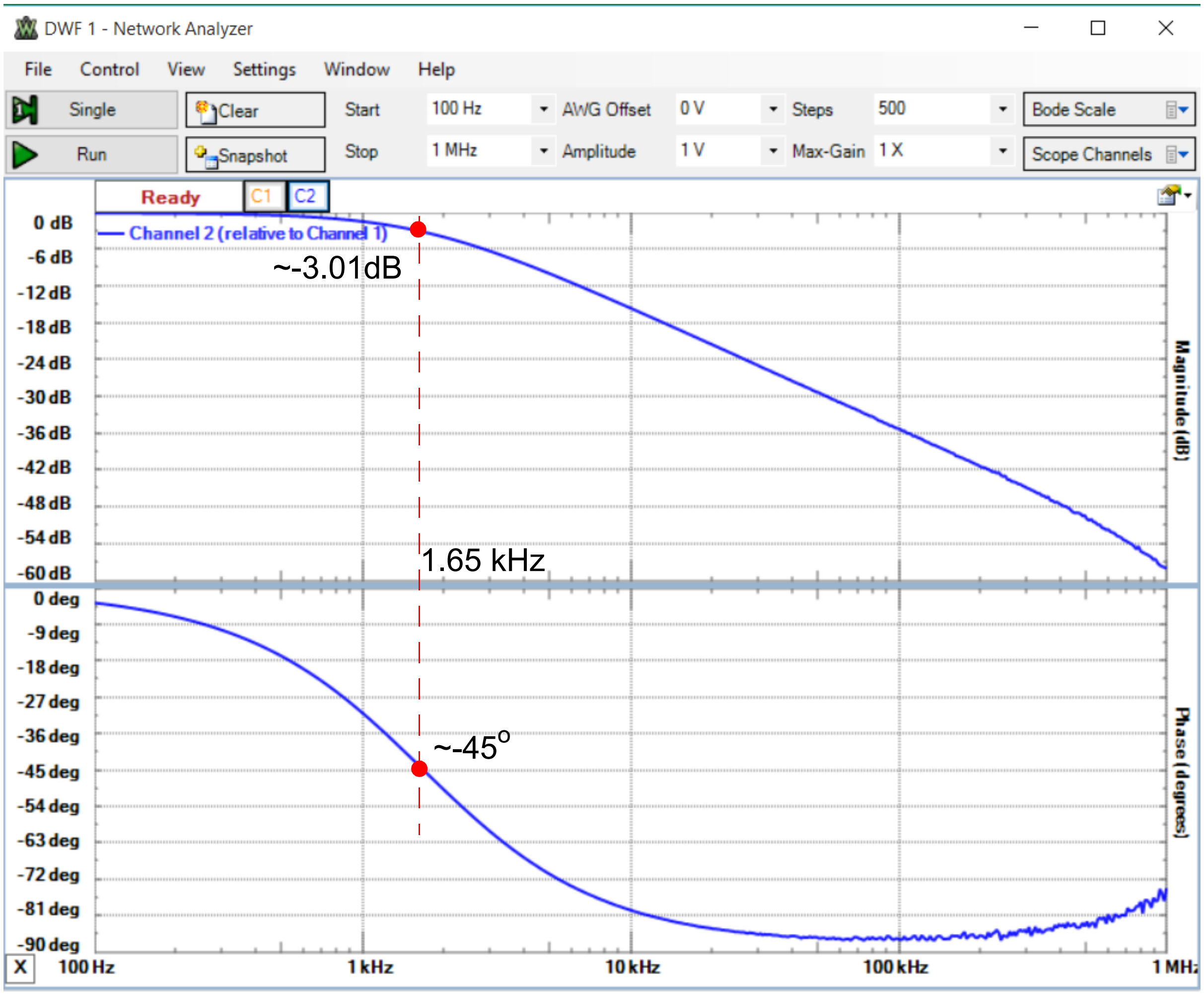
- Taking actual measurements using the Analog Discovery *network analyzer* is a great way to see how the circuit really performs
- The network analyzer is available under *More Instruments* from the Waveforms launch screen

– The network analyzer measures and displays the in-circuit measurement of $H(j2\pi f)$ between two measurement points

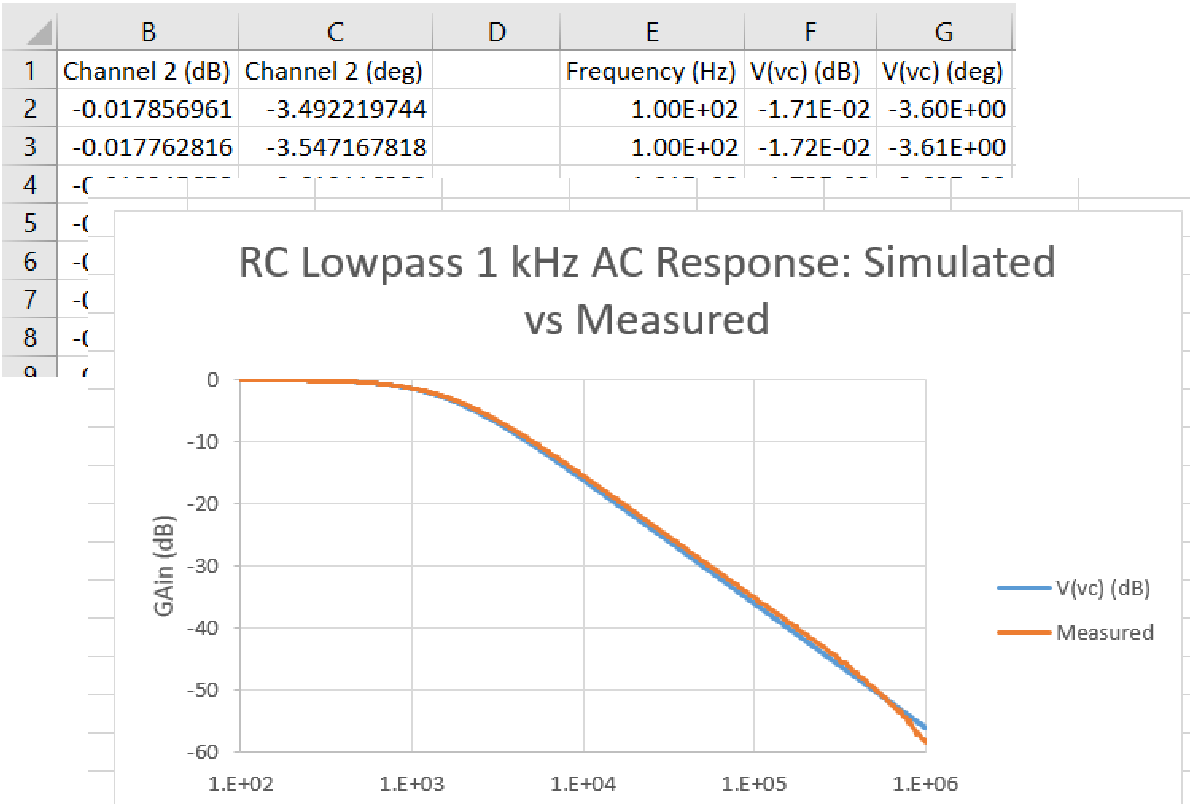
– The network analyzer takes over the scope channels and one waveform generator

– The typical configuration is to use channel one scope leads (orange) as the input and the channel two scope leads (blue) as the output

– RC lowpass filter results for $R = 1\text{k}$ and $C = 100\text{nf}$



• The LTspice and measured results can be brought together in Excel as was done for the transient response:



	B	C	D	E	F	G
1	Channel 2 (dB)	Channel 2 (deg)		Frequency (Hz)	V(vc) (dB)	V(vc) (deg)
2	-0.017856961	-3.492219744		1.00E+02	-1.71E-02	-3.60E+00
3	-0.017762816	-3.547167818		1.00E+02	-1.72E-02	-3.61E+00

– Very good agreement on the dB magnitude plots!

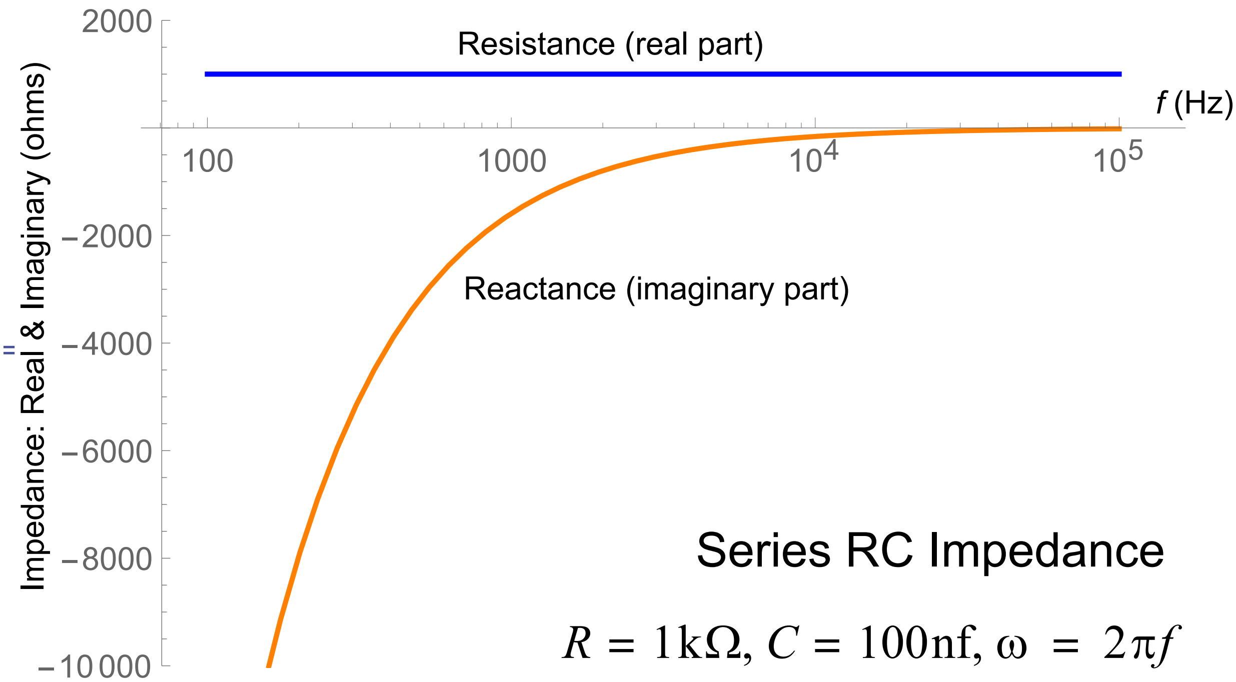
- Other calculations of interest include:
  – The total impedance looking into the RC elements

$$Z_{\text{eq}}(s) = Z_R + Z_C = R + \frac{1}{sC} \tag{6.40}$$

  – By letting $s \rightarrow j\omega$, *step 2*, the resistance and reactance components of impedance can be broken out

$$Z_{\text{eq}}(j\omega) = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C} \tag{6.41}$$

  – The plot below was obtained using Mathematica

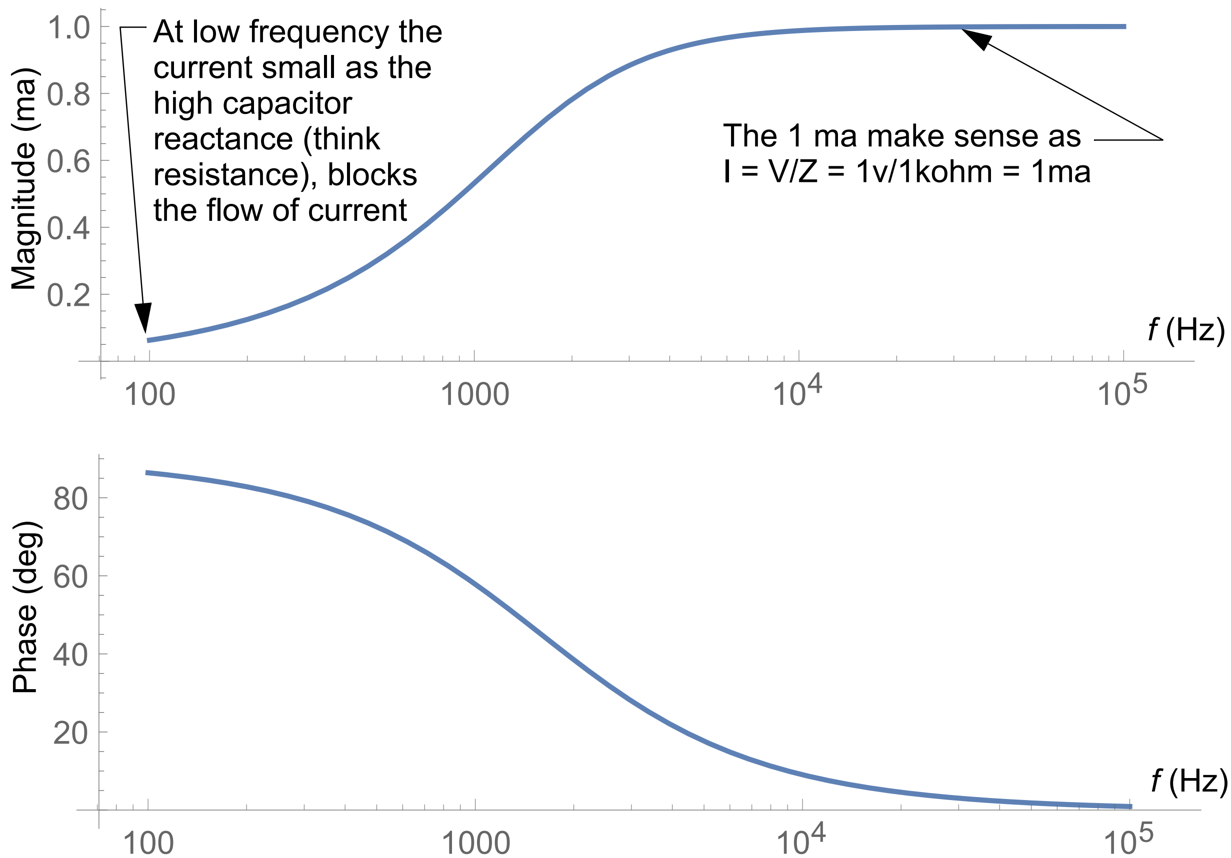


– The current flowing through the series circuit can also be calculated

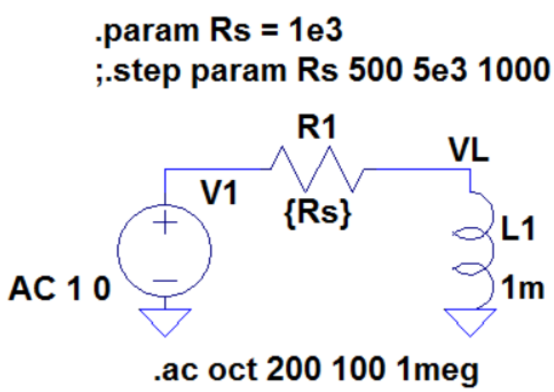
$$I = \frac{V_i}{Z_{\text{eq}}} = \frac{V_i}{R + \frac{1}{sC}} = V_i \cdot \frac{sC}{1 + sRC} \tag{6.42}$$

– Again making the substitution $\omega = j2\pi f$, you can find the current as a magnitude (in amps) and phase shift, both as a function of the operating frequency

$$I = 1\angle 0 \cdot \frac{j2\pi f}{1 + j2\pi fRC} \tag{6.43}$$



Example 6.7: RL Highpass Circuit



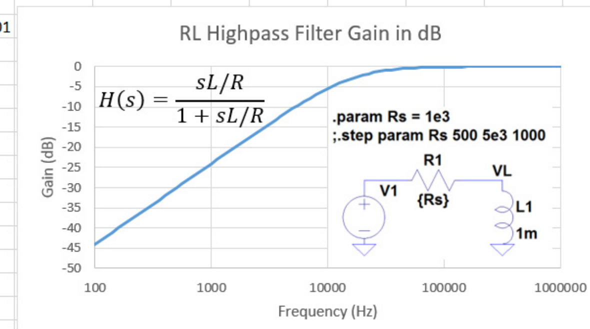
- Here I revisit the *RL* highpass circuit to consider the frequency response
- Using the voltage divider relationship it is easy to write

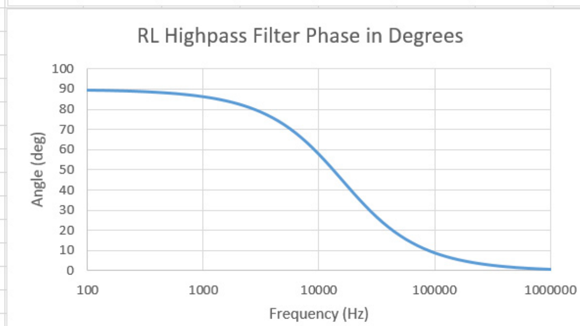
$$
\begin{aligned}
V_L(s) &= V_1(s) \cdot \frac{sL_1}{R_1 + sL_1} \\
H(s) &= \frac{V_L(s)}{V_1(s)} = \frac{sL_1}{R_1 + sL_1}
\end{aligned}
\tag{6.44}
$$

- Using Excel for the theoretical frequency response yields:

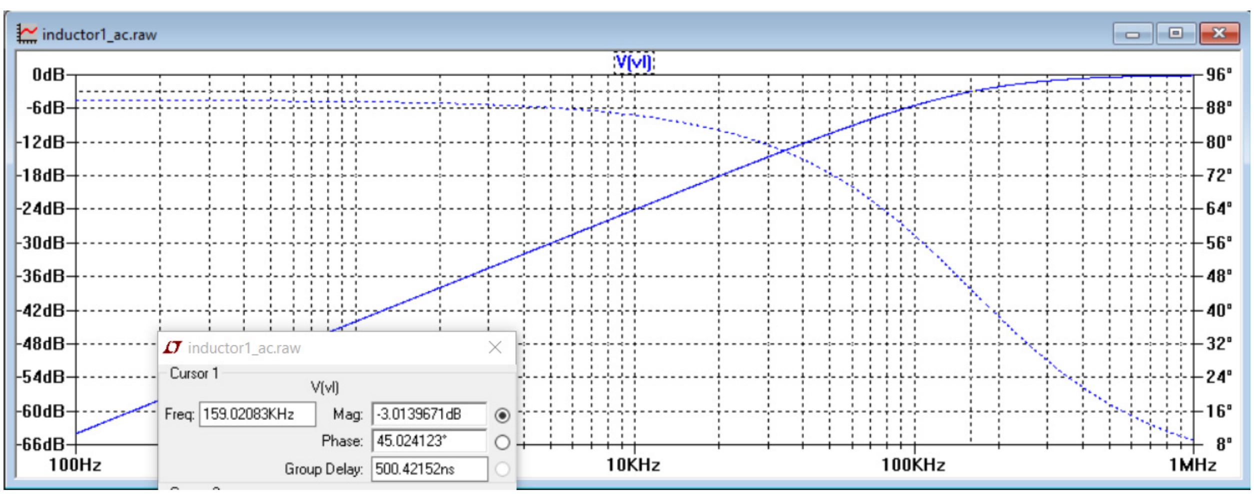
D2 =IMDIV(IMPRODUCT(C2,G2),IMSUM(1,IMPRODUCT(C2,G2)))

	A	B	C	D	E	F	G
1	x	f = 10^x	s = j*2*PI*f	H(s)	HdB(s)	Phase H(f)	L/R
2	2	100	628.318530717959j	0.0000394	-44.0366	89.64	0.00001
3	2.05	112.2018	704.984986645445j	0.0000496	-43.0366	89.59608	
4	2.1	125.8925	791.006165022012j	0.0000625	-42.0367	89.5468	
5	2.15	141.2538	887.523514621322j	0.0000787	-41.0367	89.4915	
6	2.2	158.4893	995.817762032063j	0.0000991	-40.0368	89.42946	
7	2.25	177.8279	1117.32590612166j	0.0001248	-39.0369	89.35985	
8	2.3	199.5262	1253.66028613816j	0.0001571	-38.0371	89.28174	
9	2.35	223.8721	1406.6299764725j	0.0001978	-37.0373	89.19411	
10	2.4	251.1886	1578.26479197648j	0.0002490	-36.0375	89.0958	
11	2.45	281.8383	1770.84222237266j	0.0003134	-35.0378	88.98549	
12	2.5	316.2278	1986.91765315922j	0.0003946	-34.0381	88.86173	
13	2.55	354.8134	2229.35827402299j	0.0004967	-33.0386	88.72288	
14	2.6	398.1072	2501.38112470457j	0.0006252	-32.0391	88.56711	
15	2.65	446.6836	2806.59578316113j	0.0007870	-31.0398	88.39236	
16	2.7	501.1872	3149.05226247286j	0.0009906	-30.0407	88.19632	
17	2.75	562.3413	3533.2947520559j	0.0012468	-29.0418	87.97641	
18	2.8	630.9573	3964.421916295j	0.0015691	-28.0432	87.72974	
19	2.85	707.9458	4448.15455072215j	0.0019747	-27.045	87.45307	
20	2.9	794.3282	4990.91149349751j	0.0024847	-26.0472	87.14279	
21	2.95	891.2509	5599.89479949198j	0.0031260	-25.05	86.79484	
22	3	1000	6283.18530717959j	0.0039323	-24.0535	86.40473	
23	3.05	1122.018	7049.84986645445j	0.0049454	-23.0579	85.96741	
24	3.1	1258.925	7910.06165022013j	0.0062180	-22.0635	85.47729	
25	3.15	1412.538	8875.23514621322j	0.0078154	-21.0705	84.92815	
26	3.2	1584.893	9958.17762032063j	0.0098191	-20.0793	84.31313	
27	3.25	1778.279	11173.2590612166j	0.0123302	-19.0903	83.62464	
28	3.3	1995.262	12536.6028613816j	0.0154734	-18.1041	82.85434	



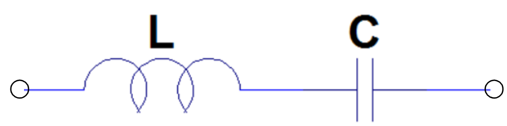


- No surprise, LTspice gets the same results effortlessly



Resonance and LC Tank Circuits

- For radio applications you need to make bandpass filtering functions for the RF front-end and IF signal processing
- Bandpass frequency response functions are also needed for building the local oscillator that can be tuned using a variable capacitor
- The key to all of this is *resonance*, which is what occurs when an inductor and capacitor are placed in the same circuit and the **reactance values cancel**
- You know that a capacitor produces negative reactance and an inductor produces positive reactance, so canceling is a reality
  – Series LC:



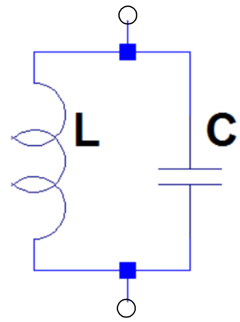
The impedance across the series connection is

$$\begin{aligned} Z_{eq}(s) &= sL + \frac{1}{sC} \\ Z_{eq}(j\omega) &= j\left[\omega L - \frac{1}{\omega C}\right] \end{aligned} \tag{6.45}$$

  – Resonance occurs when the reactance cancel each other and the effective impedance becomes zero (short circuit):

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \text{ or } f = \frac{1}{2\pi\sqrt{LC}} \tag{6.46}$$

– Parallel LC:



The impedance of the parallel connection is

$$Z_{eq}(s) = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1 + s^2 LC} \tag{6.47}$$

$$Z_{eq}(j\omega) = \frac{j\omega L}{1 - \omega^2 LC} \text{ since } j^2 = -1$$

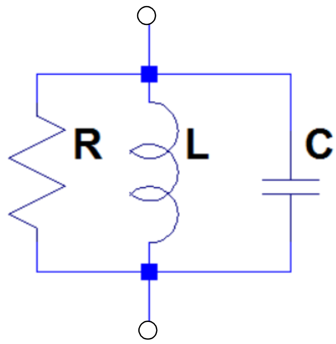
Resonance occurs when the reactance cancel each other and the effective impedance becomes infinite (open circuit):

$$\boxed{1 - \omega^2 LC = 0 \Rightarrow \omega^2 = \frac{1}{LC} \text{ or } f_o = \frac{1}{2\pi\sqrt{LC}}} \tag{6.48}$$

- The parallel resonance circuit is also known as a *tank circuit*
- Both circuit configurations can be used to make bandpass filters, but in simple radio circuits the parallel or tank circuit form, is the most common
- In a practical setting the inductor will contain some series resistance (conductor loss) and there will be resistive loading

due to other circuit elements surrounding the inductor

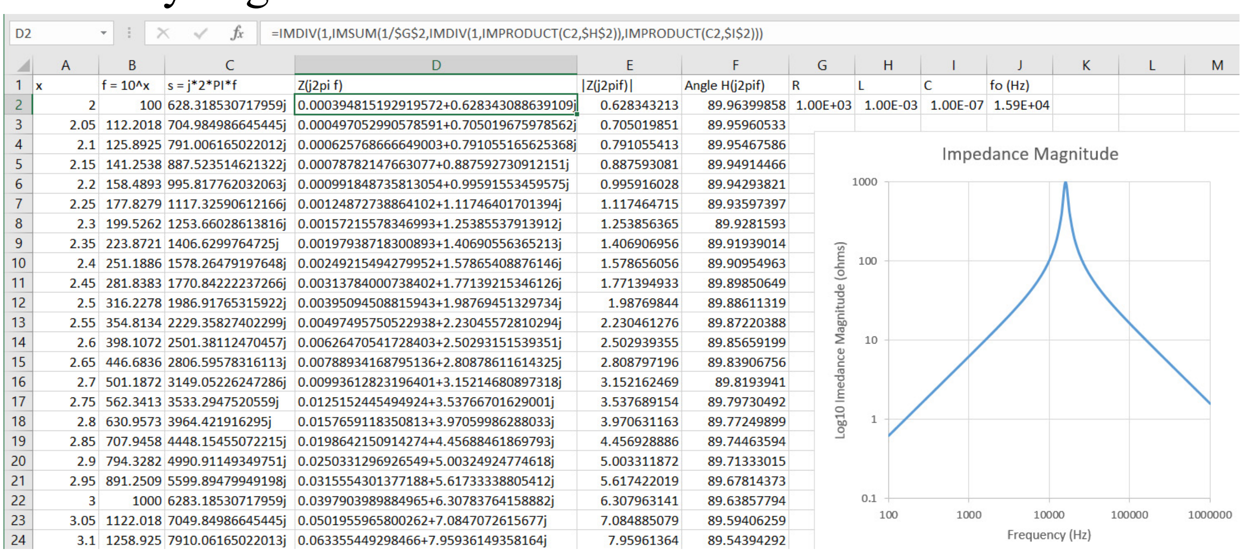
- Consider the impedance magnitude of the circuit



- The impedance magnitude is

$$\left|Z_{\text{eq}}(s)\right| = \left|\frac{1}{\frac{1}{R}+\frac{1}{sL}+sC}\right| \tag{6.49}$$

- Analyzing in Excel



D2 =IMDIV(1,IMSUM(1/G2,IMDIV(1,IMPRODUCT(C2,H2)),IMPRODUCT(C2,I2)))

	A	B	C	D	E	F	G	H	I	J
1	x	f = 10^x	s = j*2*PI*f	Z(j2pi f)	|Z(j2pif)|	Angle H(j2pif)	R	L	C	fo (Hz)
2	2	100	628.318530717959j	0.000394815192919572+0.628343088639109j	0.628343213	89.96399858	1.00E+03	1.00E-03	1.00E-07	1.59E+04
3	2.05	112.2018	704.984986645445j	0.000497052990578591+0.705019675978562j	0.705019851	89.95960533				
4	2.1	125.8925	791.006165022012j	0.000625768666649003+0.791055165625368j	0.791055413	89.95467586				
5	2.15	141.2538	887.523514621322j	0.00078782147663077+0.887592730912151j	0.887593081	89.94914466				
6	2.2	158.4893	995.817762032063j	0.000991848735813054+0.99591553459575j	0.995916028	89.94293821				
7	2.25	177.8279	1117.32590612166j	0.00124872738864102+1.11746401701394j	1.117464715	89.93597397				
8	2.3	199.5262	1253.66028613816j	0.00157215578346993+1.25385537913912j	1.253856365	89.9281593				
9	2.35	223.8721	1406.6299764725j	0.00197938718300893+1.40690556365213j	1.406906956	89.91939014				
10	2.4	251.1886	1578.26479197648j	0.00249215494279952+1.57865408876146j	1.578656056	89.90954963				
11	2.45	281.8383	1770.84222237266j	0.00313784000738402+1.77139215346126j	1.771394933	89.89850649				
12	2.5	316.2278	1986.91765315922j	0.00395094508815943+1.98769451329734j	1.98769844	89.88611319				
13	2.55	354.8134	2229.35827402299j	0.00497495750522938+2.23045572810294j	2.230461276	89.87220388				
14	2.6	398.1072	2501.38112470457j	0.00626470541728403+2.50293151539351j	2.502939355	89.85659199				
15	2.65	446.6836	2806.59578316113j	0.00788934168795136+2.80878611614325j	2.808797196	89.83906756				
16	2.7	501.1872	3149.05226247286j	0.00993612823196401+3.15214680897318j	3.152162469	89.8193941				
17	2.75	562.3413	3533.2947520559j	0.0125152445494924+3.53766701629001j	3.537689154	89.79730492				
18	2.8	630.9573	3964.421916295j	0.0157659118350813+3.97059986288033j	3.970631163	89.77249899				
19	2.85	707.9458	4448.15455072215j	0.0198642150914274+4.45688461869793j	4.456928886	89.74463594				
20	2.9	794.3282	4990.91149349751j	0.0250331296926549+5.00324924774618j	5.003311872	89.71333015				
21	2.95	891.2509	5599.89479949198j	0.0315554301377188+5.61733338805412j	5.617422019	89.67814373				
22	3	1000	6283.18530717959j	0.0397903989884965+6.30783764158882j	6.307963141	89.63857794				
23	3.05	1122.018	7049.84986645445j	0.0501955965800262+7.0847072615677j	7.084885079	89.59406259				
24	3.1	1258.925	7910.06165022013j	0.063355449298466+7.95936149358164j	7.95961364	89.54394292				

- Resonant circuits exhibit a selectivity property, that is at frequencies near resonance the impedance, or voltage, or current remain close to the value at resonance

- In the parallel RLC circuit the impedance at resonance is R

- As the frequency moves off resonance, the impedance magnitude drops
- The 3dB bandwidth of the resonator is defined as the frequency band corresponding to the 3dB or 0.707 points on the curve
- For $R = 1000$ (as in the figure above) this occurs at 707.1Ω
- The ratio of center frequency f_o divided by BW is a constant known as Q, i.e.

$$Q = \frac{f_o}{\text{BW}} \tag{6.50}$$

- At resonance you know that $X_c = -X_L$ and the so-called *loaded circuit* $Q_L = Q$ is given by

$$Q = \left.\frac{R}{X_L}\right|_{f = f_o} \tag{6.51}$$

- Rewriting in terms of 3dB BW,

$$\text{BW} = \frac{f_o}{Q} = \frac{f_o \cdot X_L}{R} = 2\pi f_o^2 \cdot \frac{L}{R} \tag{6.52}$$

- For the circuit values used above,

$$\text{BW} = \frac{2\pi(1.59\times 10^4)^2 \cdot 1\times 10^{-3}}{1000} = 1588.5\,\text{Hz} \tag{6.53}$$

Unloaded Q and Inductor Losses:

- A physical inductor has losses, which are most easily represented as a small resistor in series with L

References

[1] The ARRL Handbook for Radio Amateurs, ARRL – The National Association for Amateur Radio, 77th Edition, 2000.

[2] R.M. Mersereau and J.R. Jackson, *Circuit Analysis A Systems Approach*, Prentice Hall, 2006.

[3] F. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetic*, 7th ed, Prentice Hall, New Jersey, 2015.