Lab 1

Signals and Systems Measurements in the Frequency Domain

The first part of this experiment will serve as an introduction to the use of the FieldFox spectrum analyzer (SA) capability in making calibrated measurements of the spectrum of a signal. From your background in signals and systems you might be thinking the Fourier transform/Fourier series representation of a signal. Secondly you will work with linear time-invarient filters and measure the frequency response. Being that these measurements will be in the context of radio frequency (RF) signals and systems, you will also be exposed to a 50 ohm impedance environment. The signals considered will be common periodic signals which are produced by a high precision function generator, the Keysight 33622A (i.e., initially sine and square waveforms).

Next frequency response measurements will be investigated using the FieldFox RF/microwave network analyzer capability. In circuit theory you first learn about *two-port networks*. The primary interest here is the frequency response magnitude in dB, but phase and group delay are also of interest.

Spectrum Analysis and the Spectrum Analyzer (SA)

Communications engineering relies on the use of a spectrum analyzer to see the spectrum of signals involved in a particular system under study. When dealing with deterministic signals the primary signal type is a *periodic power signal*, such as sums of sinusoids and periodic signals having a Fourier series representation.

For periodic signals the spectrum analyzer displays a one-sided line spectrum with the lines located at the frequency of the particular harmonic component and spectral height as power in Watts delivered to the $Z_0 = 50$ ohm impedance load provided by the analyzer. The spectrum is termed the *power spectral density* (PSD). It is customary for the spectral height to have units of *dBm*, which is power in dB relative to 1 mW. For a simple sinusoid

$$x(t) = A\cos(2\pi f_0 t) \tag{1}$$

driving the analyzer 50 ohm termination/load, the analyzer will display a spectral line at f_0 Hz having power level

$$P_x = \frac{A^2}{2 \times 50}\ \text{W} \tag{2}$$

The $A^2/2$ comes from squaring the RMS value of the sinusoid which you should recall is the peak value divided by $\sqrt{2}$. The 50 comes from the power delivered to an $R_L = 50$ ohm load being V_{RMS}^2/R_L.

Power Spectrum of a Sum of Sinusoids

For the general case of M sinusoids of arbitrary amplitude, phase, and frequency, we have in math form

$$x(t) = \sum_{m=1}^{M} A_m \cos(2\pi f_m t + \phi_m) \tag{3}$$

Assuming that all f_m are distinct (if not then first combine using phasor addition), the power spectrum viewed on the spectrum analyzer is

$$S_x(f) = \sum_{m=1}^{M} \frac{A_m^2}{2 \times 50} \delta(f - f_m) \tag{4}$$

Here we are using delta functions to theoretically locate the spectral lines. The height of the delta functions, as seen on the analyzer, are of course the power level in dBm.

Observe that $x(t)$ as described above, may or may not be periodic, depending upon how the f_m are related. If the f_m are harmonically related, i.e., $f_m = m \times f_0$, where f_0 is the fundamental frequency, then $S_x(f)$ is still valid.

Power Spectrum of Square and Triangle Waveforms

When getting started with the spectrum analyzer it is instructive to consider other common periodic signals, such as the square wave and the triangle wave of Figure 1.

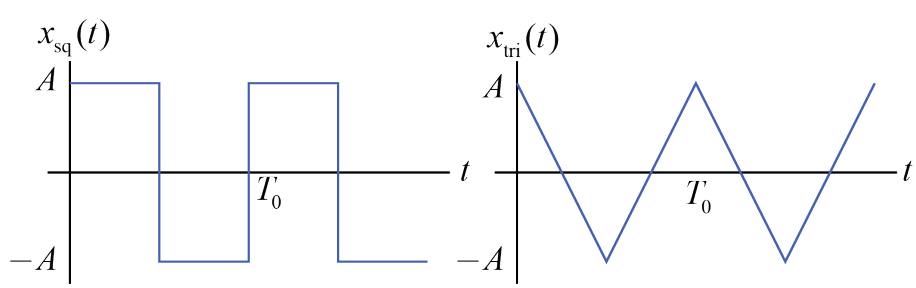


Figure 1: $\pm A$ amplitude square wave and triangle waveforms.

The complex exponential Fourier series coefficients for these waveforms is

$$X_n^{\mathrm{sq}} = \begin{cases} \frac{2A}{j\pi n}, & n = \pm 1, \pm 3, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$X_n^{\mathrm{tri}} = \begin{cases} \frac{4A}{\pi^2 n^2}, & n = \pm 1, \pm 3, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

In terms of the Fourier series coefficients, the spectrum analyzer displays the one-sided PSD having form

$$S_x(f) = \frac{X_0^2}{50}\delta(f) + \sum_{n=1}^{\infty} 2\frac{|X_n|^2}{50}\delta(f - nf_0), \tag{7}$$

where $f_0 = 1/T_0$ is the fundamental frequency. Note the power in a complex sinusoid delivered to $50\ \Omega$ is just $|X_n|^2/50$ W since $(1/T_0)\int_{T_o} |\exp(j2\pi n f_0 t)|^2\, dt = 1$. For real signals $|X_n| = |X_{-n}|$, thus when combining the positive and negative frequency terms we finally get $2|X_n|^2/50$ W.

From Parseval's theorem [1] we can also relate the total signal power to the sum of the powers in the harmonic components:

$$P_{x,\mathrm{total}} = \frac{1}{T_0}\int_{T_0} |x(t)|^2\, dt \overset{\text{also}}{=} \frac{|X_0|^2}{50} + \sum_{n=1}^{\infty} 2\frac{|X_n|^2}{50}. \tag{8}$$

This is important to understand when a signal is created to have a total power of say 1W and then you wish to know the power in the individual harmonics. It should be clear that the total power in say a square wave or triangle wave is easiest to calculate in the time domain. For the square wave in particular the integration is trivial. You confront this in Problem 4.

Network Analysis and the Vector Network Analyzer (VNA)

Subsystems that interact with communications waveforms need to be characterized. Network analysis provides frequency response information about these systems. The typical approach is to measure input and output magnitude and phase of a sinusoidal signal swept over a range of frequencies. In signals and systems the fundamental quantity of interest is the system function $H(s)$, with $s = j2\pi f$. The present focus is analyzing bandpass and lowpass filters, constructed of lumped elements, that will be used in later labs.

At high frequencies transmission effects require that these systems reside in a *doubly terminated* environment, that is the source driving the system has impedance Z_0 and the load receiving the signal also has impedance Z_0. Modeling the frequency response requires that the input/output impedance environment be considered as shown in Figure 2.

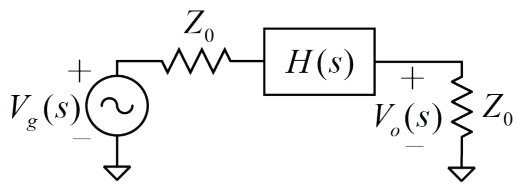


Figure 2: Obtaining the frequency response relative to $V_0(s)$ and $V_g(s)$ with and without $H(s)$ present.

The effective system function for the doubly terminated environment is the ratio of $V_0(s)$ with the filter block present divided by $V_0(s)$ with the filter block removed. Note in particular that with the filter block removed the source load combination is a simple voltage divider with $V_0(s) = V_g(s)/2$. Furthermore it is then convenient to set $V_g(j2\pi f) = 1\angle 0$. Finally, the frequency response can be obtained by setting $s = j2\pi f$ in $H(s)$. This gives

$$H(s) = \frac{V_0(j2\pi f)}{V_g(j2\pi f)/2}\bigg|_{V_g = 1\angle 0} = 2V_0(j2\pi f) \tag{9}$$

This will be explored in the LC Filter Design and Analysis subsection. Next up we consider how actual measurements are made in the lab using S–parameters capable test equipment.

RF/Microwave Network Analysis Using S–Parameters

In circuit analysis *two–port* networks form a special network type that has an input and an output. In communications signal processing we have need for linear filter two–ports so that unwanted signals can be rejected and desired or *signals–of–interest* can be passed through. Before considering specific filter design we first consider how network analysis is done at high frequencies, in particular when working in a controlled impedance environment, such as $Z_0 = R_0 = 50$ ohms. We consider Z_0 real, since that is typically the case in test and measurement, so in Figure 3 below we see a two–port embedded in a source and load terminated environment of $R_0 = 50$ ohms.

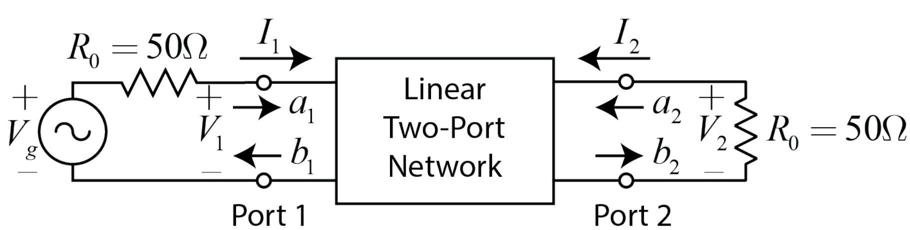


Figure 3: Two–port linear network showing voltage and current relationships along with the normalized incident and reflected wave variables a_i and b_i for $i = 1, 2$.

From the development of transmission line theory in ECE 3110, the total voltage and current, V_i and I_i, at ports $i = 1, 2$ respectively, can be written in terms of incident and reflected voltage waves, V_i^+ and V_i^- (assuming zero distance from the port) as

$$V_i = V_i^+ + V_i^- \tag{10}$$
$$I_i = \frac{V_i^+}{Z_0} - \frac{V_i^-}{Z_0} \tag{11}$$

With the S–parameters normalized incident and reflected wave variables are formed by dividing V_i^+ and V_i^- by $\sqrt{Z_0} = \sqrt{R_0}$ and define $a_i = V_i^+/\sqrt{Z_0}$ and $b_i = V_i^-/\sqrt{Z_0}$. Finally we can write

$$V_i = \left(a_i + b_i\right)\sqrt{Z_0} \tag{12}$$
$$I_i = \frac{\left(a_i - b_i\right)}{\sqrt{Z_0}} \tag{13}$$

The *scattering parameters* or s–parameters define the interaction of the normalized wave variables with the two–port at a particular frequency f in the frequency domain, that is using phasor notation where all quantities take on complex values:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}}_{\mathbf{S}} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{14}$$

The formal definition of the S–parameters is given in Table 1.

Table 1: The s–parameters for a linear two–port network.

Definition	Description
$S_{11} = \left.\frac{b_1}{a_1}\right\vert_{a_2=0}$	Input reflection coefficient with output match terminated (zero reflection).
$S_{12} = \left.\frac{b_1}{a_2}\right\vert_{a_1=0}$	Reverse transmission coefficient with input match terminated (zero reflection).
$S_{21} = \left.\frac{b_2}{a_1}\right\vert_{a_2=0}$	Forward transmission coefficient with output match terminated (zero reflection).
$S_{22} = \left.\frac{b_2}{a_2}\right\vert_{a_1=0}$	Output reflection coefficient with output match terminated (zero reflection).

Important S–Parameter Relationships

For the purposes of this lab and future labs involving network analysis, the S–parameters are a means to an end. The *end* is obtaining important system characterizations such as the frequency response of a filter or RF component in terms magnitude and phase, and perhaps the *return–loss*, i.e., reflected power from the input or output port of a device. Table 2 summarizes measurements of interest, and in particular measurements that are performed by the FieldFox N9914A we have in the lab. We assume that the S–parameters are measured as a function of frequency, so here we include f in the notation.

Table 2: RF communication measurement quantities of interest.

Measured Quantity	Description
$P_i^+ = \frac{\lvert V_i^+\rvert^2}{2Z_0} = \lvert b_i\rvert^2$, $i = 1, 2$	Average power of the incident signal/wave at ports 1 and 2
$P_i^- = \frac{\lvert V_i^-\rvert^2}{2Z_0} = \lvert b_i\rvert^2$, $i = 1, 2$	Average power of the reflected signal/wave at ports 1 and 2
$\lvert S_{11}(f)\rvert^2$	Power reflected from the network input over the power incident on the network input.
$\lvert S_{22}(f)\rvert^2$	Power reflected from the network output over the power incident on the network output.
$\lvert S_{21}(f)\rvert^2$	Power delivered to a Z_0 output load over the power available from a Z_0 source, also known as the *transducer power gain* with a Z_0 load and source.
$\lvert S_{12}(f)\rvert^2$	Reverse transducer power gain with Z_0 load and source. On passive symmetrical networks this will be the same, in theory, as $\lvert S_{21}(f)\rvert^2$.

For characterizing filters we are most interested in $S_{21}(f)$ as it gives use the gain, typically given in dB and phase of a filter or device under test. Also of interest is the S_{11}, especially $10\log\left[|S_{11}|^2\right]$ as it is the return loss (with a minus sign included) of the device under test. For a filter we want/expect a small return loss in the frequency band of interest, as this tells that the signal is passing through the device and making its way to the load, as opposed to being reflected from the input.

LC Analog Filter Design and Analysis

Many different technologies can be used for filter design. One approach to passive filter design is by the so-called *insertion loss method* [2]. The steps in the design process are shown in Figure 4.

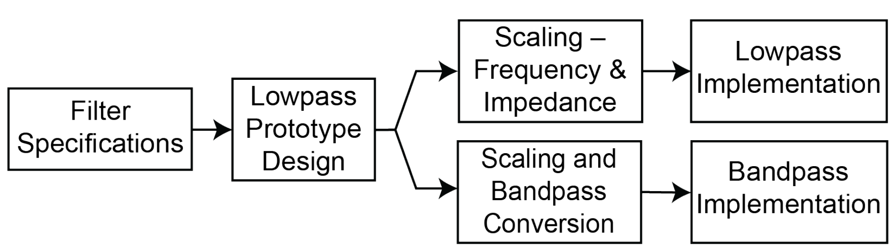


Figure 4: Analog filter design by the insertion loss method.

The filter design specifications are typically given in terms a desired amplitude response, see for example Figure 5 where the passband amplitude tolerance is $-\epsilon_{\rm dB} \leq |H(f)|_{\rm dB} \leq 0$ for $f_1 \leq f \leq f_2$ and the stopband tolerance is $|H(f)|_{\rm dB} \leq -A_s$ for $f \leq f_{s1}$ and $f \geq f_{s2}$.

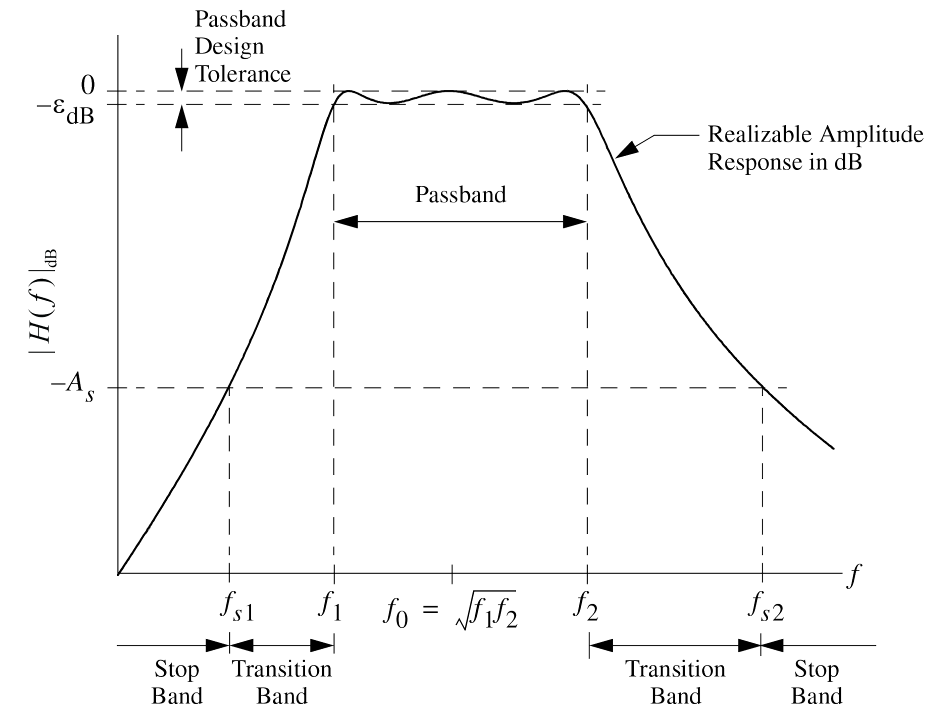


Figure 5: The frequency response of a realizable bandpass filter showing how amplitude response specifications are satisfied by a particular design.

Behavioral Level Modeling

You will see shortly that a component level design is synthesized directly from amplitude response requirements. A communication systems engineer may want a behavioral level filter model for use in analysis and simulation. Two popular amplitude response characteristics are *Butterworth* or *maximally flat* and *Chebyshev* or *equal–ripple* [2] and [3]. Chebyshev is shown in Figure 5 and is what is used in the RF Board designs. For both response types the s–domain system function is of the form

$$H(s) = \frac{b_0 + b_1 s + \cdots + b_N s^N}{a_0 + a_1 s + \cdots + a_N s^N} \tag{15}$$

where N is the filter order. By setting $s = j2\pi f$ the frequency response can be obtained. Given $H(f)$, which is equivalent to $S_{21}(f)$ from the previous section, you can then obtain the magnitude response, $|H(f)|$, magnitude response in dB, $10\log_{10}[|H(f)|^2]$, the phase response $\theta(f) = \angle H(f)$, and finally the *group delay*,

$$T_g(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df} \tag{16}$$

which has units of seconds. Group delay is a measure of the delay distortion introduced by the filter. Constant group delay is acceptable, as all frequencies passing through the filter received the same delay. Butterworth and Chebyshev filters have non-constant group delay as we shall see shortly.

The *scipy stack* provides a means to obtain the $\{b_n\}$ and $\{a_n\}$ coefficient arrays using functions found in `scipy.signal`. Relevant design functions for Butterworth Chebyshev analog filters are listed in Table 3.

Table 3: System function and frequency response functions in `scipy.signal` for analyzing Butterworth and Chebyshev filters.

Function	Description
`b, a = signal.butter(N, Wn, btype='low', analog=False, output='ba')`	Design an Nth-order digital or analog Butterworth filter and return the filter coefficient arrays `b` and `a`.
`b, a = signal.cheby1(N, rp, Wn, btype='low', analog=False, output='ba')`	Design an Nth-order digital or analog Chebyshev type I (ripple in the passband) filter and return the filter coefficient arrays `b` and `a`.
`signal.freqs(b, a, worN=200, plot=None)`	Given the M-order numerator `b` and N-order denominator `a` of an analog filter, compute its frequency response
`skdsp_comm.sigsys.splane(b, a, auto_scale=True]`	Create an s-plane pole-zero plot As input the function uses the numerator and denominator s-domain system function coefficient ndarrays `b` and `a` respectively. Assumed to be stored in descending powers of s.
`Tg = grp_delay_s(H,f)`	Group delay from frequency response. This function is presently defined in the Lab 1 sample notebook. `H` is the complex frequency response with corresponding frequency array `f`. The units of T_g are s.

Example: System Level Design of the RF Board 88-108 MHz Bandpass Filter

Design $H_{\text{but}}(s)$ and $H_{\text{cheb}}(s)$ in terms of $\{b_n\}$ and $\{a_n\}$ coefficient arrays:

```python
b_but, a_but = signal.butter(3, 2*pi*array([87e6,109e6]), btype='bandpass', analog=True, output='ba')
b_cheb, a_cheb = signal.cheby1(3, 0.5, 2*pi*array([87e6,109e6]), btype='bandpass', analog=True, output='ba')
print(b_cheb)
print(a_cheb)
```

```
[1.89031812e+24 0.00000000e+00 0.00000000e+00 0.00000000e+00]
[1.00000000e+00 1.73190256e+08 1.15244960e+18 1.31566119e+26
 4.31446976e+35 2.42736133e+43 5.24706521e+52]
```

Plot the results:

```python
f = arange(70, 130, .01) # In MHz
w, H_but = signal.freqs(b_but,a_but,2*pi*f*1e6)
w, H_cheb = signal.freqs(b_cheb,a_cheb,2*pi*f*1e6)
plot(f,20*log10(abs(H_but)))
plot(f,20*log10(abs(H_cheb)))
plot([70,130],[-.5,-.5],'r--'); plot([70,130],[-3,-3],'r--');
plot([87,87],[-30,1],'r--'); plot([109,109],[-30,1],'r--');
ylim([-30,1])
title(r'88-108 MHz Frequency Response: $N=3$ Butter vs Cheby')
ylabel(r'Gain (dB)')
xlabel(r'Frequency (MHz)')
legend((r'Butterworth',r'Chechshev'))
grid();
```

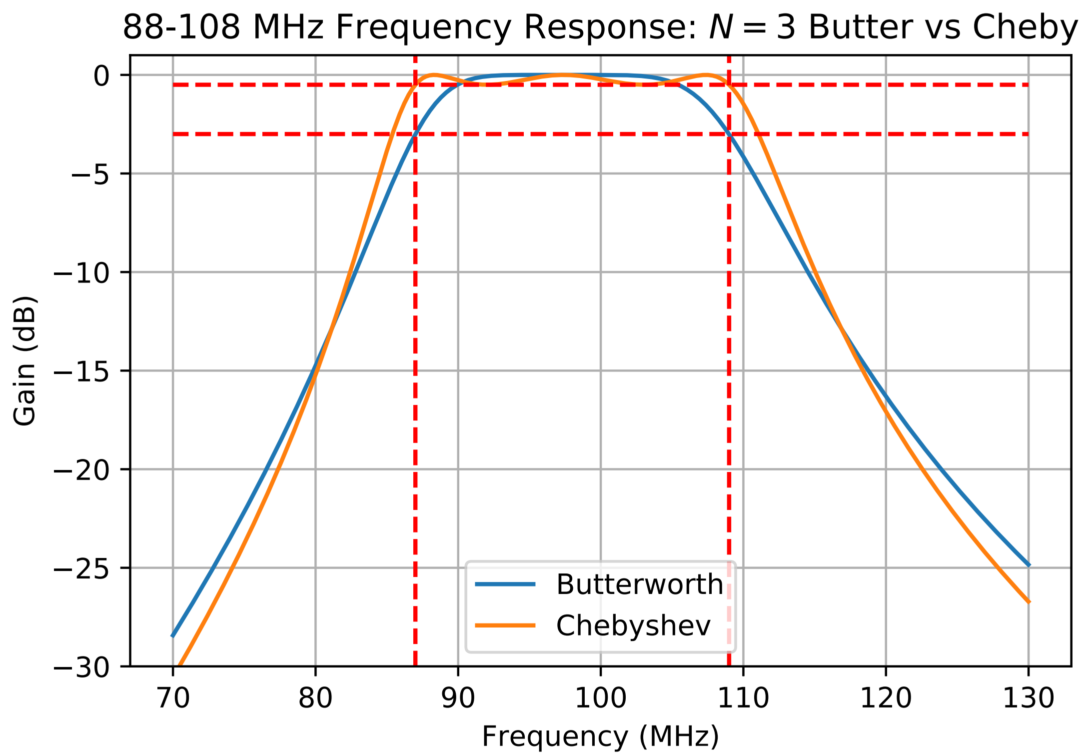


Figure 6: Frequency response magnitude in dB of Butterworth and Chebyshev bandpass filters obtained using a behavioral level model of the analog filter.

Compare the phase response:

```
plot(f,angle(H_but)*180/pi)
plot(f,angle(H_cheb)*180/pi)
title(r'88-108 MHz Frequency Response: $N=3$ Butter vs Cheby')
ylabel(r'Phase (deg)')
xlabel(r'Frequency (MHz)')
legend((r'Butterworth',r'Chebyshev'))
grid();
```

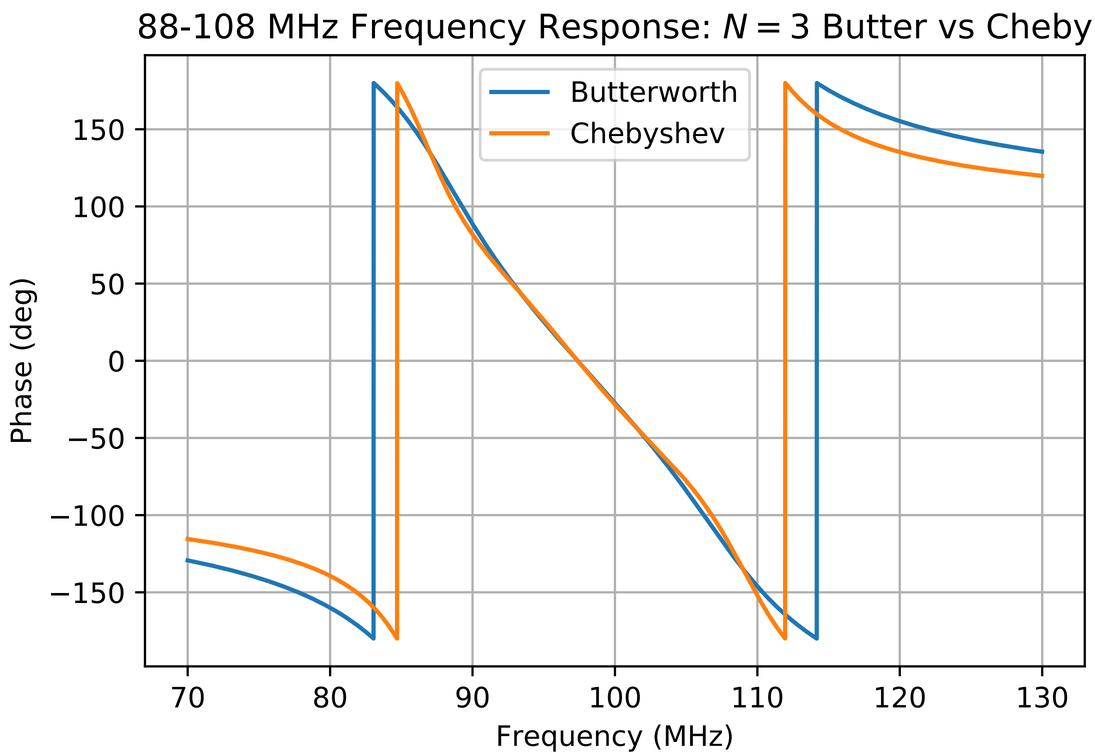


Figure 7: Frequency response phase of Butterworth and Chebyshev bandpass filters obtained using a behavioral level model of the analog filter.

Compare the group delay response:

```
Tg_but = grp_delay_s(H_but,f*1e6) # frequency in Hz
Tg_cheb = grp_delay_s(H_cheb,f*1e6)
plot(f[:-1],Tg_but*1e9) # frequency axis needs to be reduced by 1 index
plot(f[:-1],Tg_cheb*1e9)
title(r'88-108 MHz Frequency Response: $N=3$ Butter vs Cheby')
ylabel(r'Group Delay (ns)')
xlabel(r'Frequency (MHz)')
legend((r'Butterworth',r'Chebyshev'))
grid();
```

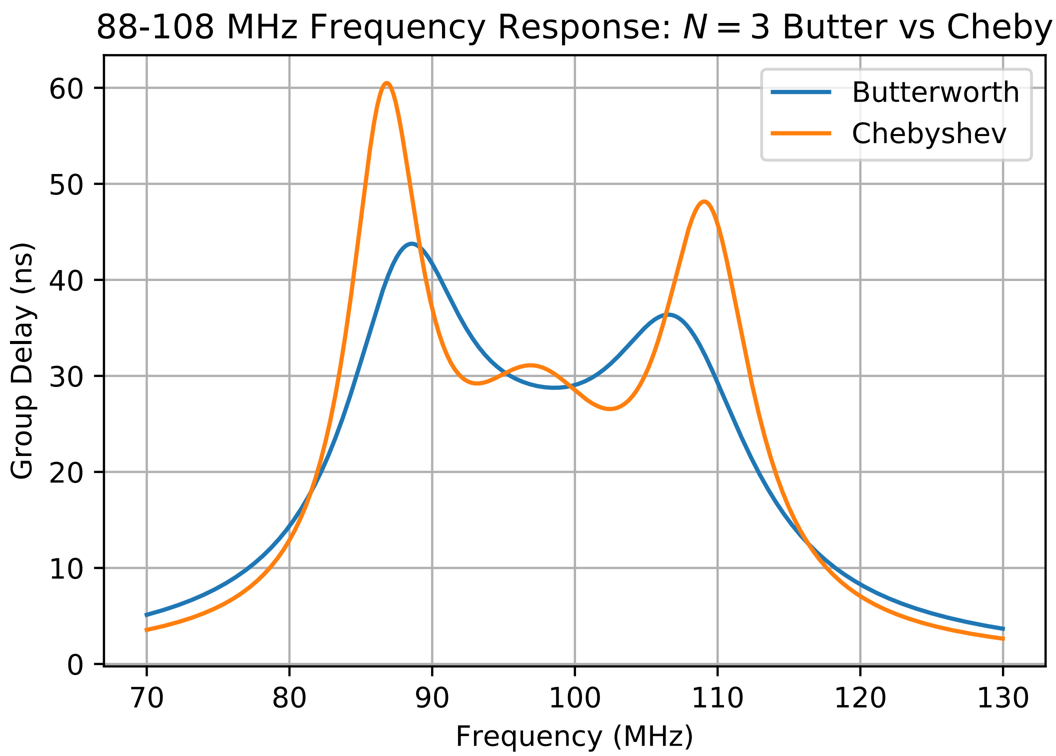


Figure 8: Frequency response group delay of Butterworth and Chebyshev bandpass filters obtained using a behavioral level model of the analog filter.

Lowpass Prototype

From these filter design specifications comes a passive inductor (L) capacitor (C) lowpass filter prototype. The filter is called a prototype because it generally will have a lowpass cutoff frequency of 1 radian/second and a normalized impedance level of one ohm. Lumped lowpass prototypes relevant to this project are of the form of a ladder network as shown in Figure 9, where the filter order is given by N.

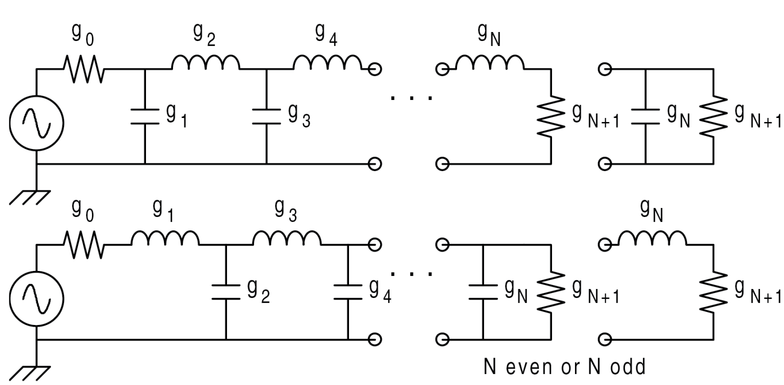


Figure 9: Prototype lowpass filter LC ladder networks, in two equivalent forms, for making both bandpass and lowpass filters.

The element values are ohms (Ω), henries (H), or farads (F), as given by the element type. The element g_0 represents the source resistance and the element g_{N+1} represents the load resistance. Filters of this type are said to be *doubly terminated*, since they require both a source resistance and load resistance for proper operation.

The element values g_1 to g_N are chosen to give a desired shape to the lowpass frequency response. Two popular shapes are *Butterworth* which has a maximally flat passband and *Chebyshev* which has an equal–ripple passband. The Chebyshev has the advantage of the Butterworth is that by allowing passband ripple the transition from passband to stopband is smaller for the same filter order N.

Butterworth

In a Butterworth lumped element design the normalized g_k values are given by [3]

$$g_0 = g_{N+1} = 1 \text{ ohm} \tag{17}$$

$$g_k = 2\sin\left[\frac{(2k-1)\pi}{2N}\right], k = 1, 2, \ldots, N \tag{18}$$

With this design the 3dB cutoff frequency is at $\omega_c = 1$ rad/s or $f_c = \omega_c/(2\pi)$.

Chebyshev

In a Chebyshev lumped element design the normalized g_k values are given by [3]

$$g_0 = 1 \text{ ohm} \tag{19}$$

$$g_{N+1} = \begin{cases} 1, & N \text{ odd} \\ 1 + 2\epsilon^2 + 2\epsilon\sqrt{1+\epsilon^2}, & N \text{ even} \end{cases} \tag{20}$$

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}, k = 2, 3, \ldots, N \tag{21}$$

where

$$a_k = \sin\frac{2k-1}{2N}\pi \tag{22}$$

$$b_k = \sinh^2\frac{\beta}{2N} + \sin^2\frac{k\pi}{N} \tag{23}$$

$$\beta = \ln\frac{\sqrt{1+\epsilon^2}+1}{\sqrt{1+\epsilon^2}-1} \tag{24}$$

$$g_1 = \frac{2a_1}{\sinh\beta/(2N)} \tag{25}$$

and

$$\epsilon = \sqrt{10^{\epsilon_{\mathrm{dB}}/10} - 1} \tag{26}$$

The cutoff frequency is again $\omega_c = 1$ rad/s, but the filter gain is just $-\epsilon_{\mathrm{dB}}$, not 3dB, as you might think.

Scaling and Conversion

The third block of Figure 4 is scaling and conversion. In the lowpass prototype the impedance level is 1 ohm. To scale the impedance level to say R_0 ohms, multiply all resistors and inductors by R_0 and divide all capacitors by R_0. To summarize we remap the resistors, inductors, and capacitors as follows:

$$R_k \to R_0 \cdot R_k, \quad L_k \to R_0 \cdot L_k, \quad C_k \to C_k/R_0 \tag{27}$$

Under the bandpass transformation inductors become series LC circuits and capacitors become parallel LC circuits. Assuming impedance scaling has already taken place, the only variables needed besides L and C are the fractional bandwidth, w, and the center frequency, f_0. The transformation equations and the circuit element replacements are shown in Figure 11. The general topology of an $N = 3$ bandpass filter is also shown in Figure 11.

To transform to a bandpass design at a specified center frequency f_0, fractional bandwidth $w = \mathrm{BW}/f_0$, and impedance level $Z_0 = R_0$ we follow the steps of Figure 10, shown for a third order design in Figure 5.

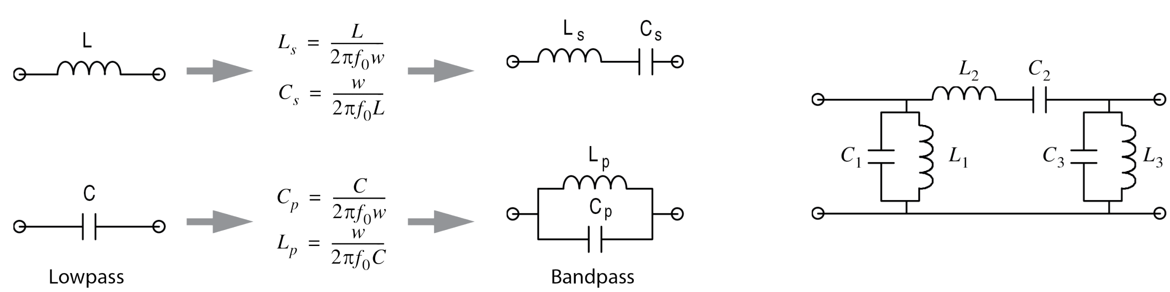


Figure 10: Lowpass to bandpass transformation for the case of a $N = 3$ bandpass design (6 poles in the bandpass).

For a lowpass design impedance scaling occurs followed by scaling to move the cutoff frequency from $\omega_c = 1$ to $2\pi f_c$. This means that

$$L_k \to L_k/(2\pi f_c), \quad C_k \to C_k/(2\pi f_c) \tag{28}$$

The upper and lower band edge frequencies f_1 and f_2 are related to the center frequency f_0 and fractional bandwidth w via

$$f_1 = \frac{f_0}{2}(\sqrt{4+w^2} - w) \tag{29}$$

$$f_2 = \frac{f_0}{2}(\sqrt{4+w^2} + w) \tag{30}$$

Network Synthesis Functions for Butterworth and Chebyshev

The sample Jupyter notebook contains bandpass and lowpass network synthesis equations as described in Table 4.

Table 4: Lumped element LC filter synthesis functions.

Function	Description
`Rg,L,C,RL = lc_butter_bpf(N,f1,f2,R)`	Synthesize a Butterworth bandpass filter into L and C values given the order N and the passband edge frequencies f_1 and f_2 or the center frequency and fractional bandwidth f_0 and $w = (f_2 - f_1)/f_0$. Note $f_0 = \sqrt{f_1 f_2}$.
`Rg,L,C,RL = lc_cheby_bpf(N,RdB,f1,f2,R)`	Synthesize a Chebyshev bandpass filter into L and C values given the order N, the ripple in dB, R_{dB}, and the passband edge frequencies f_1 and f_2 or the center frequency and fractional bandwidth f_0 and $w = (f_2 - f_1)/f_0$. Note $f_0 = \sqrt{f_1 f_2}$. and for even order designs $R_g \neq R_L$.
`Rg,L,C,RL = lc_butter_lpf(N,f1,f2,R)`	Synthesize a Butterworth lowpass filter into L and C values given the order N and the passband edge frequencies f_1 and f_2 or the center frequency and fractional bandwidth f_0 and $w = (f_2 - f_1)/f_0$. Note $f_0 = \sqrt{f_1 f_2}$.
`Rg,L,C,RL = lc_cheby_lpf(N,RdB,f1,f2,R)`	Synthesize a Chebyshev lowpass filter into L and C values given the order N, the ripple in dB, R_{dB}, and the passband edge frequencies f_1 and f_2 or the center frequency and fractional bandwidth f_0 and $w = (f_2 - f_1)/f_0$. Note $f_0 = \sqrt{f_1 f_2}$. and for even order designs $R_g \neq R_L$.

Example: 30 MHz $N = 3$ Chebyshev Bandpass with 5 MHz Bandwidth and R_{dB} =1.0

```
f1 = 27.5e6 # Hz
f2= 32.5e6 # Hz
Rg,L_BPF30,C_BPF30,RL = lc_cheby_bpf(3,1.0,f1,f2,50)
```

```
*****  f0 =   2.99e+07 Hz and w = 0.167 *****
```

```
L_BPF30*1e9 # nH
```

```
array([  21.99991455, 1582.16317954,   21.99991455])
```

```
C_BPF30*1e12 # pF
```

```
array([1288.25908705,   17.91319012, 1288.25908705])
```

The element values obtained in this example will be used

Network Sensitivity Analysis and Inductor *Q*–Factor Loss

In practical design the element values will be different from theory and there will be loss, primarily due to the *quality factor*, Q, of real inductors. In this subsection we discuss how this can be modeled, so that the difference between measured results and ideal theory can be better understood. Additionally all the element values have tolerance associated with them. The RF Board mica chip capacitors have a tolerance of 5%, but the chip inductors have tolerance of 10% up to 20%.

Finite Q

The *unloaded Q* associated with a series resonant circuit is defined as [3]

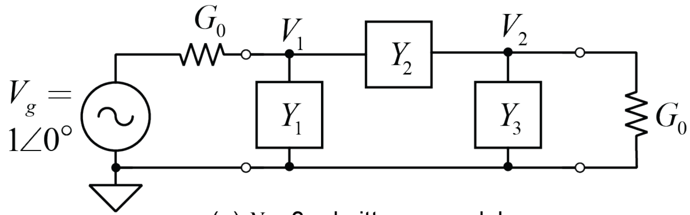
$$Q_u = \frac{\omega_0 L}{R_s} \quad \text{or} \quad R_s = \frac{\omega_0 L}{Q_u} \tag{31}$$

where $\omega_0 = 2\pi f_0$ and the approximate value for Q_u can be found in the chip inductor data sheet. Similarly for a parallel resonant circuit [3]

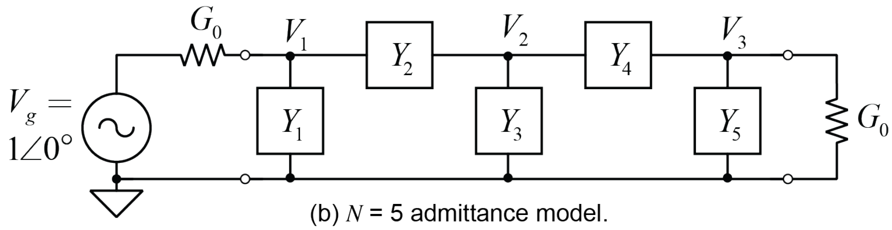
$$Q_u = \frac{R_p}{\omega_0 L} \quad \text{or} \quad R_p = \omega_0 L Q_u \tag{32}$$

An Admittance Model for Ladder Network Frequency Response

In this subsection we derive an expression for the frequency response of the $N = 3$ and $N = 5$ ladder networks shown in Figure 11.



(a) N = 3 admittance model.



(b) N = 5 admittance model.

Figure 11: Modeling the $N = 3$ and $N = 5$ LC ladder networks using admittances so that component variation and loss due to unloaded Q can be represented.

The admittance values Y_i can hold combinations of RLC values for series and parallel resonant circuits of bandpass filters that incorporate loss, i.e., R_s or R_p and C and RL values found in lowpass filters. The analysis objective is met by solving for $V_2(j2\pi f)$ and $V_3(j2\pi f)$ with $V_g = 1\angle 0^\circ$ in Figure 11 (a) and (b) respectively.

For the $N = 3$ circuit writing out nodal equations in matrix form gives:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_0 + Y_1 + Y_2 & -Y_2 \\ -Y_2 & G_0 + Y_2 + Y_3 \end{bmatrix}^{-1} \begin{bmatrix} V_g G_0 \\ 0 \end{bmatrix} \tag{33}$$

With the aide of a symbolic solver, such as SymPy or a hand–calculation (try it)

$$H_3(s) = \frac{2G_0 Y_2}{(G_0 + Y_1 + Y_2)(G_0 + Y_2 + Y_3) - Y_2^2} \tag{34}$$

For the $N = 5$ circuit writing out the nodal equations in matrix form gives

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} G_0 + Y_1 + Y_2 & -Y_2 & 0 \\ -Y_2 & Y_2 + Y_3 + Y_4 & -Y_4 \\ 0 & -Y_4 & G_0 + Y_4 + Y_5 \end{bmatrix}^{-1} \begin{bmatrix} V_g G_0 \\ 0 \\ 0 \end{bmatrix} \tag{35}$$

This time using a symbolic solver for the 3×3 case, we find (note not trivial in this case)

$$\begin{aligned} H_5(s) = 2Y_2Y_4 \cdot \big[& Y_1(Y_2(Y_4 + Y_5 + G_0) + Y_3(Y_4 + Y_5 + G_0) + Y_4(Y_5 + G_0)) \\ & Y_2(Y_3(Y_4 + Y_5 + G_0) + Y_4(Y_5 + 2G_0) + G_0(Y_5 + G_0)) \\ & G_0(Y_3(Y_4 + Y_5 + G_0) + Y_4(Y_5 + G_0))\big]^{-1} \end{aligned} \tag{36}$$

The sample Jupyter notebook contains functions that implement the $N = 3$ and $N = 5$ ladder network frequency response, for bandpass and lowpass designs respectively. The admittances Y_i are assigned circuit element values as defined in Table 5.

Table 5: Admittance value assignments in the s–domain.

Circuit Element Type	Equation
Series LC with Loss	$Y_i = \left[\frac{1}{sC_s} + sL_s + R_s\right]^{-1}$
Parallel LC with Loss	$Y_i = sC_p + \frac{1}{sL_p} + \frac{1}{R_p}$
Series L with loss	$Y_i = \frac{1}{sL} + \frac{1}{R_s}$
Series C	$Y_i = sC$

Putting it all together, the functions for producing $H(f)$ are described in Table 6.

Table 6: Admittance model function for producing frequency response from L and C element values and unloaded Q values.

Function	Description
`H = lc6_Ymodel_bpf(f,L,C,f0,Q=[1000,1000,1000],R0=50)`	Full loss for $N = 3$ LC bandpass filter using an admittance ladder network model. `f` is the frequency value array, `L` is the inductance value array, `C` is the capacitance value array, `f0` is the center frequency or cutoff frequency for loss modeling. `Q` is the quality factor of the inductors, and `R0` is the operating characteristic impedance, e.g., 50 Ohms. By default `Q` is essentially infinite, so no loss.
`H = lc5_Ymodel_lpf(f,L,C,f0,Q=[1000,1000,1000],R0=50)`	Full loss for $N5 = LC$ lowpass filter using an admittance ladder network model. `f` is the frequency value array, `L` is the inductance value array, `C` is the capacitance value array, `f0` is the center frequency or cutoff frequency for loss modeling. `Q` is the quality factor of the inductors, and `R0` is the operating characteristic impedance, e.g., 50 Ohms. By default `Q= 1000` is essentially infinite, so no loss.

Example: 30 MHz BPF (continued)

Moving forward with the 30 MHz bandpass element values, we now compare the ideal frequency response (no loss and exact LC component values) with actual measured $S_{21}(f)$, and and the `lc6_Ymodel_bpf` with components *tweaked* and loss introduced by reducing `Q`. The Jupyter notebook widgets are utilized so *tuning* of L and C values and Q values can be used to match the tolerance and loss model output to the measured response. First the Python code, then Figure 12is a screenshot of the controls and the resulting plot.

```
p_L1 = widgets.FloatSlider(value=0.,min=-20,max=20,step=0.1,description='L1 tol in %')
p_C1 = widgets.FloatSlider(value=0.,min=-5,max=5,step=0.1,description='C1 tol in %')
p_Q1 = widgets.FloatLogSlider(value=1000.,base=10,min=1,max=3,step=0.02,description='Q1')
p_L2 = widgets.FloatSlider(value=0.,min=-10,max=10,step=0.1,description='L2 tol in %')
p_C2 = widgets.FloatSlider(value=0.,min=-5,max=5,step=0.1,description='C2 tol in %')
p_Q2 = widgets.FloatLogSlider(value=1000.,base=10,min=1,max=3,step=0.02,description='Q2')
p_L3 = widgets.FloatSlider(value=0.,min=-20,max=20,step=0.1,description='L3 tol in %')
p_C3 = widgets.FloatSlider(value=0.,min=-5,max=5,step=0.1,description='C3 tol in %')
p_Q3 = widgets.FloatLogSlider(value=1000.,base=10,min=1,max=3,step=0.02,description='Q2')
uiV1 = widgets.VBox([p_L1,p_C1,p_Q1])
uiV2 = widgets.VBox([p_L2,p_C2,p_Q2])
uiV3 = widgets.VBox([p_L3,p_C3,p_Q3])
ui = widgets.HBox([uiV1,uiV2,uiV3])

def f_BPF(p_L1,p_C1,p_Q1,p_L2,p_C2,p_Q2,p_L3,p_C3,p_Q3):
    # Bring some variables into this function as globals to make the input arg
    # list shorter; other approaches work too, but this was an easy fix
    global L_BPF30, C_BPF30, f, S21magdB
    #f = arange(20,40,.01)*1e6
    # Tune the L and C values away from ideal and
    L_tol_err = array([(1+p_L1/100)*L_BPF30[0],(1+p_L2/100)*L_BPF30[1],(1+p_L3/100)*L_BPF30[2]])
    C_tol_err = array([(1+p_C1/100)*C_BPF30[0],(1+p_C2/100)*C_BPF30[1],(1+p_C3/100)*C_BPF30[2]])
    H_ideal = lc6_Ymodel_bpf(f,L_BPF30,C_BPF30,30e6,Q=[1000,1000,1000])
    H_loss = lc6_Ymodel_bpf(f,L_tol_err,C_tol_err,30e6,Q=[p_Q1,p_Q2,p_Q3])
    plot(f/1e6,20*log10(abs(H_ideal)))
    plot(f/1e6,20*log10(abs(H_loss)))
    plot(f/1e6,S21magdB)
    #semilogx(f,20*log10(abs(H)))
    ylabel(r'Gain (dB)')
```

```python
    xlabel(r'Frequency (MHz)')
    title(r'30 MHz $N=3$ BPF')
    legend((r'Ideal Model',r'Loss/Tol. Model',r'Measured'))
    grid();

output = widgets.interactive_output(f_BPF,{'p_L1': p_L1, 'p_C1': p_C1, 'p_Q1': p_Q1,
                                            'p_L2': p_L2, 'p_C2': p_C2, 'p_Q2': p_Q2,
                                            'p_L3': p_L3, 'p_C3': p_C3, 'p_Q3': p_Q3})
display(ui, output)
```

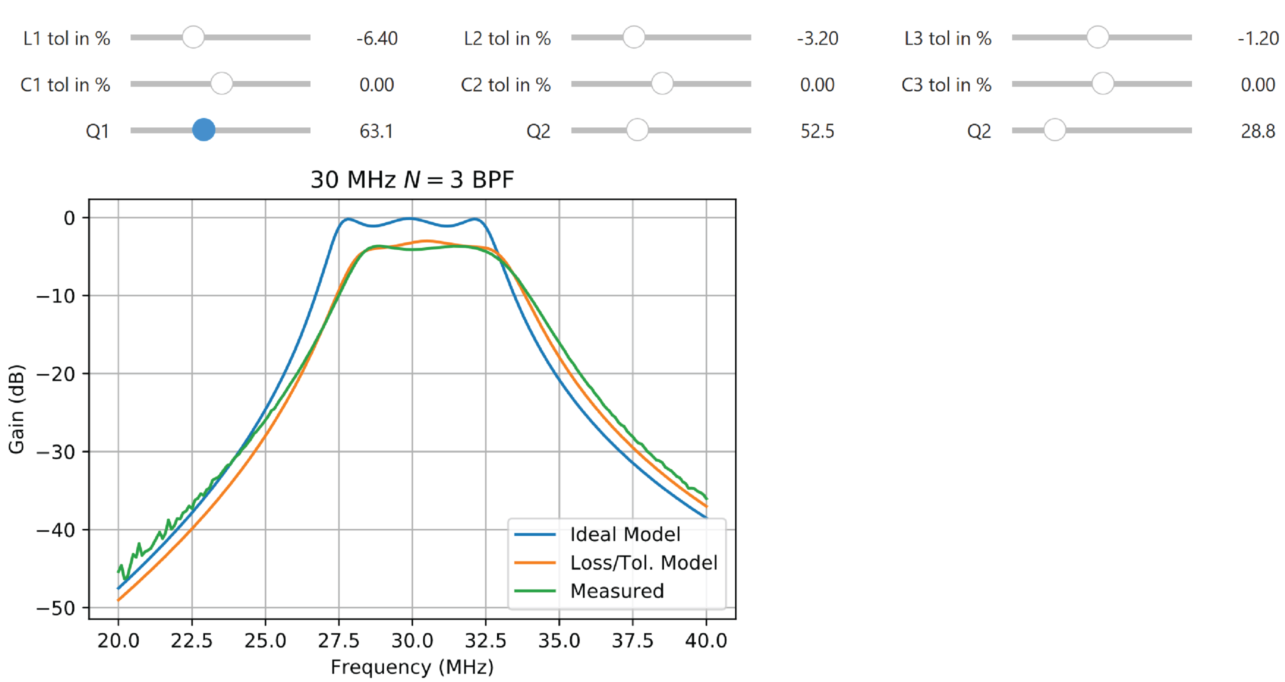


Figure 12: 30 MHz bandpass filter, theory, loss/tolerance model *tweaked*, and the actual measured response.

To continue this example consider group delay of ideal theory, the loss model with L value tweaking and loss, and the measured S_{21} group delay. The currently installed network analyzer options on the N9914A FieldFox, do not provide group delay (see the FieldFox N9914A instrument specs in the next section), but we can make this computation in the Jupyter notebook using `grp_delay_s()`. First we must combine the measured magnitude and phase data to form the complex frequency response, `H_meas`.

- Create a tolerance array, `L_tol`, that multiplies the ideal L values to produce shifted values; for tolerances we use the values found using the GUI slider adjustments made earlier in this example.

The Python code found in the Lab 1 sample notebook is shown first, followed by Figure 13 which shows the group delay comparison plots.

```
# Tweak L values by -6.4%, -3.2%, and -1.2%
L_tol = array([1.0-0.064,1.0-0.032,1.0-0.012])
```

```
# Convert measured S_21 from dB & angle to complex freq resp.
H_meas = 10**(S21magdB/10)*exp(1j*S21deg*pi/180)
H_ideal = lc6_Ymodel_bpf(f,L_BPF30,C_BPF30,sqrt(f1*f2),Q=[1000,1000,1000])
Tg_H_ideal = grp_delay_s(H_ideal,f)
H_loss = lc6_Ymodel_bpf(f,L_BPF30*L_tol,C_BPF30,sqrt(f1*f2),Q=[100,30,50])
Tg_H_loss = grp_delay_s(H_loss,f)
Tg_H_meas = grp_delay_s(H_meas,f)
plot(f[:-1]/1e6,Tg_H_ideal*1e9)
plot(f[:-1]/1e6,Tg_H_loss*1e9)
plot(f[:-1]/1e6,Tg_H_meas*1e9)
ylim([0, 350])
xlim([23,38])
xlabel(r'Frequency (MHz)')
ylabel(r'Group delay (ns)')
title(r'30 MHz Bandpass Filter Frequency Response - $T_g(f)$')
legend((r'Ideal Model',r'Loss/Tol. Model',r'Measured'))
grid();
```

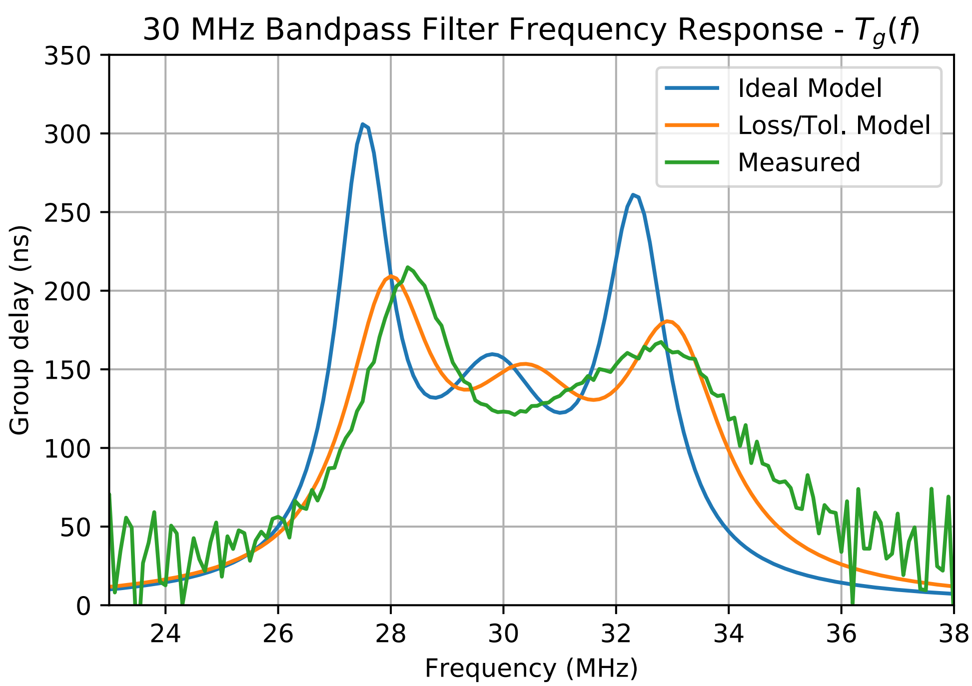


Figure 13: Group delay comparison of theory, L value tweaked with loss model, and measured comparison.

The lower band edge group delay peaks are similar in magnitude for the Loss/Tol model and measured. The upper band edge peaks are close. The *noise* in the measured group delay is not too surprising, given that differentiating to form T_g will add high high frequency measurement noise.

Keysight FieldFox (model N9914A)

The Keysight FieldFox N9914A is a portable combination analyzer. In the words of the product literature combination = Cable and antenna tester (CAT) + Vector network analyzer (VNA) + Spectrum analyzer (SA). The various interfaces to analyzer are shown the photographs of Figure 14.

Figure 14: The Keysight N9914A spectrum/network analyzer, showing the from panel and side and top connections.

For this first lab the SA and VNA (NA) modes of the N9914A FieldFox of the most importance. Extracts from some the specification tables can found in Tables 7–121. Table 7 give a brief overview of performance by model number.

Table 7: Keysight FieldFox Specs in brief.

Specifications in Brief

See the FieldFox Handheld Analyzer Data Sheet for a complete listing of the specifications:
http://literature.cdn.keysight.com/litweb/pdf/5990-9783EN.pdf.
Cable and antenna tester are referred to as CAT and vector network analyzer is referenced as VNA in this section.

Model	CAT and VNA frequency	Spectrum analyzer frequency[1]	Test port connectors
RF & microwave (combination) analyzers			
N9913A	30 kHz to 4 GHz	100 kHz to 4 GHz	Type-N (f)
N9914A	30 kHz to 6.5 GHz	100 kHz to 6.5 GHz	Type-N (f)
N9915A	30 kHz to 9 GHz	100 kHz to 9 GHz	Type-N (f)
N9916A	30 kHz to 14 GHz	100 kHz to 14 GHz	Type-N (f)
N9917A	30 kHz to 18 GHz	100 kHz to 18 GHz	Type-N (f)
N9918A	30 kHz to 26.5 GHz	100 kHz to 26.5 GHz	3.5 mm (m)
N9950A	300 kHz to 32 GHz	9 kHz to 32 GHz	NMD 2.4 mm (m)
N9951A	300 kHz to 44 GHz	9 kHz to 44 GHz	NMD 2.4 mm (m)
N9952A	300 kHz to 50 GHz	9 kHz to 50 GHz	NMD 2.4 mm (m)

Notes:
1. Usable to 5 kHz.
2. Order Option 100 for 3.5 mm (m) test port connectors. With N9938A-100, the spectrum analyzer is built with 3.5 mm test port connectors instead of the standard Type-N (f). Option 100 is a prerequisite for Option 320 for N9938A.

Tables 8 and 9 drill down on the network analyzer (NA) capability.

Table 8: Keysight FieldFox Part 1 of NA capability.

Cable and antenna analyzer (CAT) and vector network analyzer (VNA)

The performance listed in this section applies to the cable and antenna analyzer (referred to as CAT) and vector network analyzer (referred to as VNA) capabilities.

Model	N9913/ 14/ 15/ 16/ 17/ 18A N9925/ 26/ 27/ 28A	N9950 /51 /52A
Measurements		
CAT	Distance-to-fault (dB), return loss, VSWR, DTF (VSWR), cable loss (1 port), optional insertion loss (2 port), DTF (linear), DTF / return loss dual display	
TDR cable measurements	TDR (rho), TDR (ohm), DTF / TDR	
VNA T/R	S11, S21 and insertion loss	
VNA full 2 port	S11, S21, S22, S12 mag and phase, VSWR, linear, phase, Smith chart, polar, group delay, unwrapped phase, real/imaginary	
Calibration types	CalReady, 1-port OSL, frequency response, enhanced response, QSOLT, unknown thru 2-port, Ecal, QuickCal (not available in N995xA models)	
Number of traces	4	
Number of markers	6	
Marker functions	Peak, minimum, target, bandwidth measurement with Q, marker tracking	
Data points	101, 201, 401, 601, 801, 1001 ,1601, 4001, 10,001	
Frequency reference: -10 to 55 °C		
Accuracy	± 0.7 ppm (spec) + aging ± 0.4 ppm (typical) + aging	
Accuracy, when locked to GPS	± 0.01 ppm (spec)	

Table 9: Keysight FieldFox Part 2 of NA capability.

Cable and antenna analyzer (CAT) and vector network analyzer (VNA) (continued)

Model	N9913/ 14/ 15/ 16/ 17/ 18A N9925/ 26/ 27/ 28A		N9950 /51 /52A	
Power level accuracy (typical)				
	± 1.5 dB at -15 dBm for frequencies > 250 kHz		± 0.7 dB at −15 dBm, for frequencies > 500 kHz to 10 MHz ± 0.5 dB at −15 dBm, for frequencies > 10 MHz to 50 GHz	
Power step size				
	Flat power, in 1 dB steps, is available across the whole frequency span (nominal)			
Distance to fault				
Range	Range = velocity factor x speed of light x (number of points -1) / frequency span x 2			
	Number of points auto coupled according to start and stop distance entered.			
Range resolution	Resolution = range / (number of points -1)			
System dynamic range [1, 2] : Port 1 or Port 2, high power, 300 Hz IF bandwidth, -10 to 55 °C				
Frequency	Spec	Typical	Spec	Typical
> 300 kHz to 9 GHz[3]	95 dB	100 dB	—	—
> 9 to 14 GHz	91 dB	97 dB	—	—
> 14 to 18 GHz	90 dB	94 dB	—	—
> 18 to 20 GHz	87 dB	90 dB	—	—
> 20 to 25 GHz	74 dB	79 dB	—	—
> 25 to 26.5 GHz	65 dB	70 dB	—	—
> 300 kHz to 1 MHz	—	—	—	70 dB (nominal)
> 1 to 10 MHz	—	—	—	100 dB (nominal)
> 10 MHz to 20 GHz [4]	—	—	100 dB	110 dB
> 20 to 44 GHz[5]	—	—	90 dB	100 dB
> 44 to 50 GHz [6]	—	—	81 dB	90 dB
Trace noise [7]: Port 1 or Port 2, high power, 300 Hz IF bandwidth, spec, -10 to 55 °C				
Frequency	Magnitude/Phase (dB rms/deg rms)			
> 300 kHz to 20 GHz	± 0.004 / ± 0.070			
> 20 to 26.5 GHz	± 0.007/ ± 0.140			
> 26.5 to 32 GHz	± 0.007/ ± 0.140			
> 32 to 50 GHz	± 0.008 / ± 0.220			
IF Bandwidth [8]				
Bandwidth	10 Hz, 30 Hz, 100 Hz, 300 Hz, 1 kHz, 3 kHz, 10 kHz, 30 kHz, 100 kHz			

Table 10, 11, and 12 cover specs associated with the spectrum analyzer (SA) capability.

Table 10: Keysight FieldFox Part 1 of SA capability.

Spectrum analyzer

The performance listed in this section applies to the spectrum analyzer capabilities.

Model	N9913 /14 /15 /16 /17 /18A N9935 /36 /37 /38A		N9950 /51 /52A N9960 /61 /62A	
Measurements				
Spectrum analyzer	Spectrum, channel power, adjacent power, occupied bandwidth, analog demodulation, tune and listen			
Number of traces	Same as network analyzer (see page 31)			
Number of markers	Same as network analyzer (see page 31)			
Interference analysis	Spectrogram, waterfall and record/playback			
Input attenuator range	0 to 30 dB, in 5 dB steps			
Frequency span	Resolution: 1 Hz			
Frequency reference: -10 to 55°C	Same as network analyzer (see page 31)			
Preamplifier	The preamplifier covers the full-band with nominal gain of 20 dB			
Tracking generator	Built in, full-band based on the model maximum frequency			
Resolution bandwidth (RBW), range (-3 dB bandwidth)				
Zero span: 10 Hz to 5 MHz: 1, 3, 10 sequence				
Non-zero span: 1 Hz to 5 MHz: 1, 1.5, 2, 3, 5, 7.5, 10 sequence				
Video bandwidth (VBW)				
1 Hz to 5 MHz in 1, 1.5, 2, 3, 5, 7.5, 10 sequence				
Total absolute amplitude accuracy Temperature (23 ± 5 ºC)				
10 dB attenuation, input signal -10 to -5 dBm, peak detector, preamplifier off, 300 Hz RBW, all settings auto-coupled, includes frequency response uncertainties. No warm-up required.				
	Spec	**Typical**	**Spec**	**Typical**
100 kHz to 18 GHz	± 0.8 dB	± 0.35 dB	–	–
> 18 to 26.5 GHz	± 1.0 dB	± 0.5 dB	–	–
> 9 to 100 kHz	–	–	± 1.6 dB	± 0.6 dB
> 100 kHz to 2 MHz	–	–	± 1.3 dB	± 0.6 dB

Table 11: Keysight FieldFox Part 2 of SA capability.

Spectrum Analyzer (Option 233 on Combination Analyzers)

Frequency and time specifications

	Models	Frequency range[1]	
N991xA, N993xA	N9913A	100 kHz to 4 GHz	Usable to 5 kHz
	N9914A	100 kHz to 6.5 GHz	Usable to 5 kHz
Frequency reference, -10 to 55°C			
Accuracy		± 0.7 ppm (spec) + aging	
		± 0.4 ppm (typical) + aging	
Accuracy, when locked to GPS		± 0.01 ppm (spec)	
Accuracy, when GPS antenna is disconnected		± 0.2 ppm (nominal)[2]	
Aging rate		± 1 ppm/yr for 20 years (spec), will not exceed ± 3.5 ppm	
Frequency readout accuracy (start, stop, center, marker)			
	± (readout frequency x frequency reference accuracy + RBW centering + 0.5 x horizontal resolution)		Horizontal resolution = frequency span / (trace points – 1) RBW centering: • 5% x RBW, FFT mode (nominal) • 16% x RBW, step mode (nominal)

Marker frequency counter	
Accuracy	± (marker frequency x frequency reference accuracy + counter resolution)
Resolution	1 Hz

Frequency Span	**Spec**
Range	0 Hz (zero span), 10 Hz to maximum frequency range of instrument
Resolution	1 Hz
Accuracy	± (2 x RBW centering + horizontal resolution) for detector = Normal
Sweep time readout	Measured value of the time required to complete a sweep from start to finish, including time to tune receiver, acquire data, and process trace.

Resolution bandwidth (RBW)	**Nominal**	
Range (-3 dB bandwidth)		
Zero span	10 Hz to 5 MHz	1, 3, 10 sequence
Non-zero span	1 Hz to 5 MHz	1, 1.5, 2, 3, 5, 7.5, 10 sequence < 300 kHz, 300 kHz, 1 MHz, 3 MHz, 5 MHz (Other RBWs may be set depending on settings)
		Step keys change RBW in 1, 3, 10 sequence
Selectivity (−60 dB / −3 dB)	4:1	
Video bandwidth (VBW)		
	1 Hz to 5 MHz	1, 1.5, 2, 3, 5, 7.5, 10 sequence

Table 12: Keysight FieldFox Part 3 of SA capability.

Amplitude accuracy and range specifications

Amplitude range		
Measurement range	DANL to +20 dBm	
Input attenuator range	0 to 30 dB, in 5 dB steps	
Preamplifier		**Nominal**
Frequency range	Full band (100 kHz to maximum frequency of instrument)	
Gain	N991xA, N993xA	+20 dB, 100 kHz to 26.5 GHz
	N995xA, N996xA	+20 dB, 100 kHz to 7.5 GHz +15 dB, > 7.5 to 50 GHz
Max safe input level	**Average CW power**	**DC**
N991xA, N993xA	+27 dBm, 0.5 watts	± 50 VDC
N995xA, N996xA	+25 dBm, 0.3 watts	± 40 VDC
Display range		
Log scale	10 divisions 0.01 to 100 dB/division in 0.01 dB steps	
Linear scale	10 divisions	
Scale units	dBm, dBmV, dBμV, dBmA, dBμA, W, V, A, dBμV/m, dBμA/m, dBG, dBT	

Total absolute amplitude accuracy (dB)

10 dB attenuation, input signal -15 to -5 dBm, peak detector, preamplifier off, 300 Hz RBW, all settings auto-coupled, includes frequency response uncertainties. No warm-up required.

N995xA, N996xA[2]	Spec (23 ± 5°C)	Spec (-10 to 55°C)	Typical (23 ± 5°C)	Typical (-10 to 55°C)
9 to 100 kHz	± 1.60	± 2.50	± 0.60	± 1.30
> 100 kHz to 2 MHz	± 1.30	± 1.90	± 0.60	± 0.80
> 2 to 15 MHz	± 1.00	± 1.20	± 0.30	± 0.50
> 15 MHz to 32 GHz	± 0.80	± 1.00[3]	± 0.30	± 0.50

The complete manuals are linked from the [ECE 4670 Web Site](). See the center tab accordion labeled Keysight N9914A and 33600A manuals.

Keysight 33600A (model 33622A)

The Keysight 33622A is a high precision two channel waveform generator producing carrier frequencies up to 120 MHz. Many waveform types and modulation options are support, including arbitrary waveforms input via the front panel USB port. Key features of this True*form* generator are:

- 1 GSa/s sampling rate and up to 120 MHz bandwidth
- Arbs with sequencing and up to 64 MSa memory
- 1 ps jitter, 200x better than DDS generators
- 5x lower harmonic distortion than DDS

Photographs of the front and back panels are given in Figure 15.

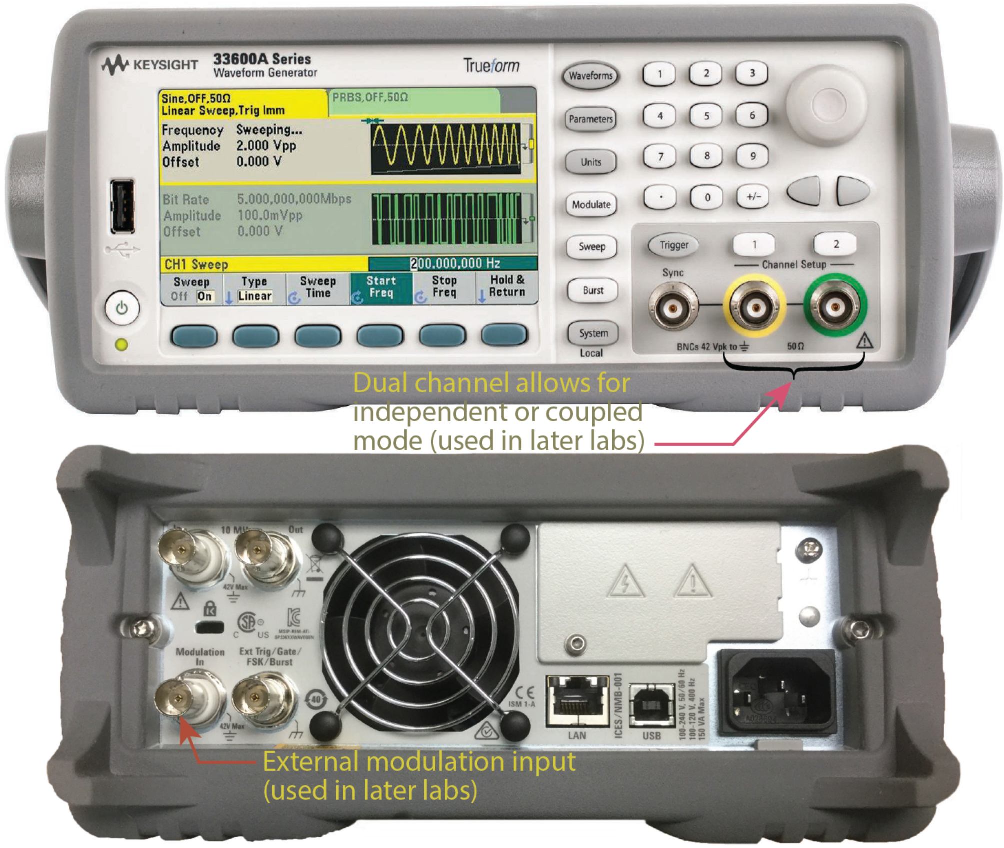


Figure 15: The Keysight 33600A signal generator, front and back.

In Lab 1 basic functionality is all that is required. In Lab 2 and beyond advanced modulation capability will be utilized along with the low jitter and dual channel capability. Tables 13–16 provide detailed specs. Table 13 gives the specs in brief.

Table 13: Keysight 33600A Specs in brief.

Instrument characteristics

Model Number in the Lab

Models & options				
Model number	33611A	33612A	33621A	33622A
Maximum frequency	80 MHz	80 MHz	120 MHz	120 MHz
Number of channels	1	2	1	2
Waveforms				
Standard	Sine, Square, Ramp, Pulse, Triangle, Gaussian Noise, PRBS (Pseudorandom Binary Sequence), DC			
Built-in arbitrary	Cardiac, Exponential Fall, Exponential Rise, Gaussian Pulse, Haversine, Lorentz, D-Lorentz, Negative Ramp, Sinc			
User-defined arbitrary	Up to 4 MSa (64 MSa with Option MEM) with multi-segment sequencing			
Operating modes & modulation types				
Operating modes	Continuous, Modulate, Frequency Sweep, Counted Burst, Gated Burst			
Modulation types	AM, FM, PM, FSK, BPSK, PWM, Sum (carrier + modulation)			

Table 14 and 15 drill down on the waveform capability of the generator.

Table 14: Keysight 33600A waveform capability part 1.

Waveform characteristics (all 33600A models)

Sine				
Frequency ranges				
$V_{OUT} \leq 10$ Vpp	1 µHz to 60 MHz, 1-µHz resolution			
$V_{OUT} \leq 8$ Vpp	1 µHz to 80 MHz, 1-µHz resolution			
$V_{OUT} \leq 4$ Vpp	1 µHz to 120 MHz, 1-µHz resolution [1]			
Amplitude flatness (rel. to 1 kHz) (spec) [2, 3]	**V_{OUT} = 1 Vpp**	**V_{OUT} > 1 Vpp**		
f_{OUT} < 10 MHz	± 0.10 dB	± 0.10 dB		
f_{OUT} = 10 MHz to 60 MHz	± 0.20 dB	± 0.25 dB		
f_{OUT} = 60 MHz to 80 MHz	± 0.30 dB	± 0.40 dB		
f_{OUT} = 80 MHz to 120 MHz [1]	± 0.40 dB	± 0.50 dB		
Harmonic distortion (typ) [2]	**V_{OUT} = 1 Vpp**	**V_{OUT} = 4 Vpp**	**V_{OUT} = 8 Vpp**	**V_{OUT}=10 Vpp**
f_{OUT} < 1 MHz	–70 dBc	–69 dBc	–68 dBc	–67 dBc
f_{OUT} < 1 MHz to 10 MHz	–61 dBc	–58 dBc	–54 dBc	–51 dBc
f_{OUT} > 10 MHz	–43 dBc	–36 dBc	–40 dBc	–39 dBc
THD (typ) [2]	**V_{OUT} = 1 Vpp**	**V_{OUT} > 1 Vpp**		
f_{OUT} = 20 Hz to 20 kHz	0.03%	0.04%		
Non-harmonic spurious ($V_{OUT} \geq$ 300 mVpp) (typ) [2, 4]				
f_{OUT} < 10 MHz	–80 dBc			
f_{OUT} = 10 MHz to 60 MHz	–75 dBc			
f_{OUT} > 60 MHz	–70 dBc			
Phase noise (SSB) (meas) [5]	**f_{OUT} = 80 MHz**	**f_{OUT} = 80 MHz, Opt OCX**	**f_{OUT} = 120 MHz [1]**	**f_{OUT}= 120 MHz, Opt OCX [1]**
100-Hz offset	–105 dBc/Hz	–114 dBc/Hz	–101 dBc/Hz	–110 dBc/Hz
1-kHz offset	–116 dBc/Hz	–122 dBc/Hz	–112 dBc/Hz	–118 dBc/Hz
10-kHz offset	–122 dBc/Hz	–125 dBc/Hz	–118 dBc/Hz	–121 dBc/Hz
100-kHz offset	–129 dBc/Hz	–131 dBc/Hz	–125 dBc/Hz	–127 dBc/Hz

Table 15: Keysight 33600A waveform capability part 2.

Waveform characteristics (all 33600A models)

Sine				
Frequency ranges				
$V_{OUT} \leq 10$ Vpp	1 µHz to 60 MHz, 1-µHz resolution			
$V_{OUT} \leq 8$ Vpp	1 µHz to 80 MHz, 1-µHz resolution			
$V_{OUT} \leq 4$ Vpp	1 µHz to 120 MHz, 1-µHz resolution [1]			
Amplitude flatness (rel. to 1 kHz) (spec) [2, 3]	**V_{OUT} = 1 Vpp**	**V_{OUT} > 1 Vpp**		
f_{OUT} < 10 MHz	± 0.10 dB	± 0.10 dB		
f_{OUT} = 10 MHz to 60 MHz	± 0.20 dB	± 0.25 dB		
f_{OUT} = 60 MHz to 80 MHz	± 0.30 dB	± 0.40 dB		
f_{OUT} = 80 MHz to 120 MHz [1]	± 0.40 dB	± 0.50 dB		
Harmonic distortion (typ) [2]	**V_{OUT} = 1 Vpp**	**V_{OUT} = 4 Vpp**	**V_{OUT} = 8 Vpp**	**V_{OUT}=10 Vpp**
f_{OUT} < 1 MHz	–70 dBc	–69 dBc	–68 dBc	–67 dBc
f_{OUT} < 1 MHz to 10 MHz	–61 dBc	–58 dBc	–54 dBc	–51 dBc
f_{OUT} > 10 MHz	–43 dBc	–36 dBc	–40 dBc	–39 dBc
THD (typ) [2]	**V_{OUT} = 1 Vpp**	**V_{OUT} > 1 Vpp**		
f_{OUT} = 20 Hz to 20 kHz	0.03%	0.04%		
Non-harmonic spurious ($V_{OUT} \geq$ 300 mVpp) (typ) [2, 4]				
f_{OUT} < 10 MHz	–80 dBc			
f_{OUT} = 10 MHz to 60 MHz	–75 dBc			
f_{OUT} > 60 MHz	–70 dBc			
Phase noise (SSB) (meas) [5]	**f_{OUT} = 80 MHz**	**f_{OUT} = 80 MHz, Opt OCX**	**f_{OUT} = 120 MHz [1]**	**f_{OUT}= 120 MHz, Opt OCX [1]**
100-Hz offset	–105 dBc/Hz	–114 dBc/Hz	–101 dBc/Hz	–110 dBc/Hz
1-kHz offset	–116 dBc/Hz	–122 dBc/Hz	–112 dBc/Hz	–118 dBc/Hz
10-kHz offset	–122 dBc/Hz	–125 dBc/Hz	–118 dBc/Hz	–121 dBc/Hz
100-kHz offset	–129 dBc/Hz	–131 dBc/Hz	–125 dBc/Hz	–127 dBc/Hz

Table 16 gives details on the modulation capability of the generator.

Table 16: Keysight 33600A modulation capability.

Modulating signals

Carrier	Sine	Square	Ramp	Triangle	Noise	PRBS	Arbitrary	External
Sine	▪	▪	▪	▪	▪	▪	▪	▪
Square & pulse	▪	▪	▪	▪	▪	▪	▪	▪
Ramp & triangle	▪	▪	▪	▪	▪	▪	▪	▪
Gaussian noise	▪	▪	▪	▪		▪	▪	▪
PRBS	▪	▪	▪	▪	▪		▪	▪
Arbitrary	▪	▪	▪	▪	▪	▪		▪

Modulation, burst, and sweep characteristics

Amplitude modulation (AM)	
Source	Internal or external (all models), or other channel (33612A/22A only)
Type	Full-Carrier or Double-Sideband Suppressed-Carrier (DSSC)
Depth [11]	0% to 120%, 0.01% resolution
Frequency modulation (FM) [12]	
Source	Internal or external (all models), or other channel (33612A/22A only)
Deviation	1 µHz to 40 MHz (33611A/12A) or 60 MHz (33621A/22A), 1-µHz resolution
Phase modulation (PM)	
Source	Internal or external (all models), or other channel (33612A/22A only)
Deviation	0° to 360°, 0.1° resolution
Frequency-shift key modulation (FSK) [12]	
Source	Internal timer or rear-panel connector
Mark & space	Any frequency within the carrier signal's range
Rate	≤ 1 MHz
Binary phase-shift key modulation (BPSK)	
Source	Internal timer or rear-panel connector
Phase shift	0° to 360°, 0.1° resolution
Rate	≤ 1 MHz
Pulse-width modulation (PWM)	
Source	Internal or external (all models), or other channel (33612A/22A only)
Deviation [6]	0% to 100% of pulse width, 0.01% resolution
Additive modulation (Sum)	
Source	Internal or external (all models), or other channel (33612A/22A only)
Ratio [11]	0% to 100% of carrier amplitude, 0.01% resolution
Burst characteristics [10]	
Type	Counted or gated
Counted burst operation	Each trigger event causes the instrument to produce from 1 to 10^8 or an "infinite" number of waveform cycles.
Gated burst operation	Instrument produces waveforms while the trigger is in the "on" state. For Gaussian Noise, waveform generation stops immediately when the trigger is in the "off" state. All other waveforms stop at the completion of a cycle; more than one cycle might elapse before generation stops.
Start/stop phase	-360° to +360°, 0.1° resolution
Trigger source	Internal timer or rear-panel connector
Marker	Indicated by the trailing edge of the Sync pulse; adjustable to any cycle of the burst.

Modulation input

Connector	Rear-panel BNC, shell and pin isolated from chassis (±42 V maximum)
Assignment	Channel 1, Channel 2, or both
Voltage level (nom)	±1 V or ±5 V full scale, selectable
Input Impedance (nom)	5 kΩ
Bandwidth (–3 dB) (typ)	0 Hz to 100 kH

The complete manuals are linked from the ECE 4670 Web Site. See the center tab accordion labeled Keysight N9914A and 33600A manuals.

Laboratory Exercises

The first three problems involve the use of the spectrum analyzer to characterize familiar signals: sine, square, and triangle waveforms. The last problems deal with frequency response characterizations of two bandpass and one lowpass filter found on the RF test board of Figure 16.

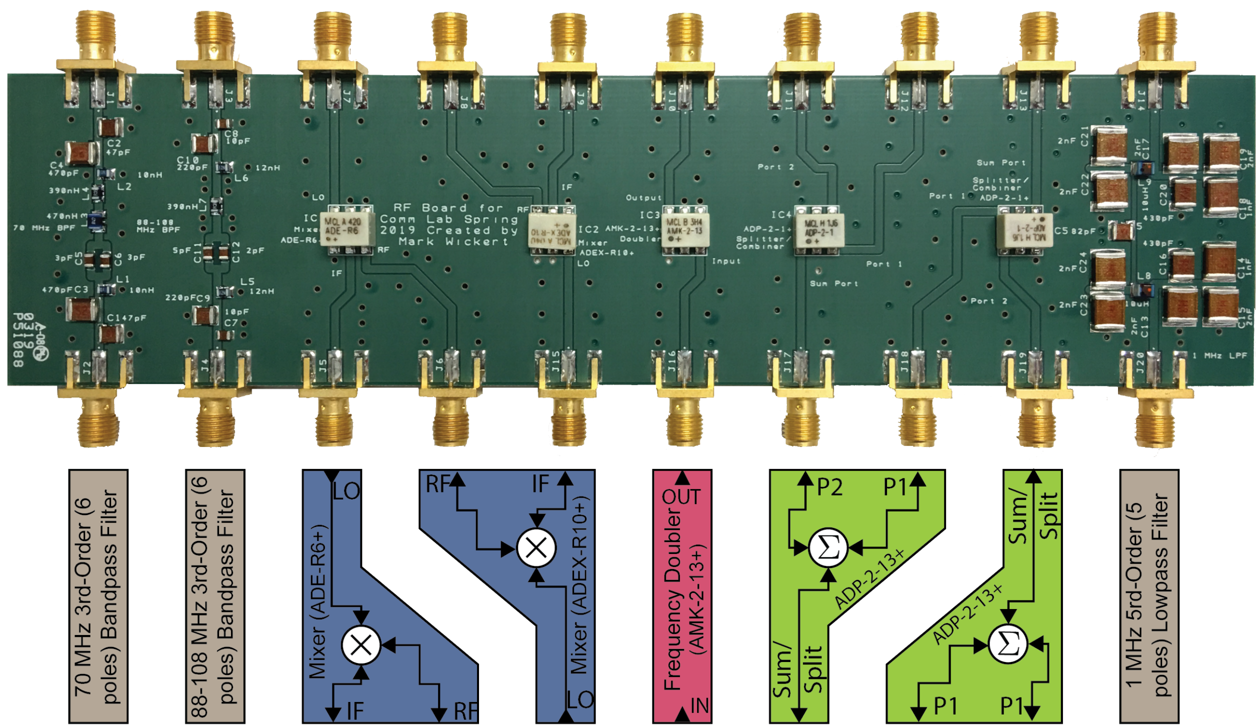


Figure 16: RF board photograph and block diagram of subsystems: 3 filters, 2 mixers, 1 doubler, 2 splitters.

The filters will be characterized in this lab. The remaining components will start being used in Lab 3 and beyond.

- The filters are used to reject unwanted signals and pass signals of interest
- The two mixers are radio frequency (RF) signal multipliers that are used to modulate, demodulate, and frequency translate signal
- The two mixers cover different RF frequency bands, both are known as double–balanced mixers (DBM)
- The frequency doubler is used to passively double the frequency of a pure carrier (sinusoid) or a sinusoid with modulation present
- The Keysight 33600A generator upper frequency limit is 120 MHz, so the doubler extends the range of carrier frequencies available for modulation and demodulation
- The two passive power splitters/combiners are used to sum two signals with 3 dB loss to each input port and can be used to split an input signal into two copies with a loss of 3 dB
- The ports of the splitter/combiner are all matched to 50 ohms and there is isolation provided between the ports

- As a signal combiner one input may be the signal of interest and the other may be noise or interference
- As a splitter one output may drive other circuitry and the other output serves as a test point
- As a splitter one output may drive the oscilloscope and one drive the spectrum analyzer

Problem 1

Using the Keysight 33600A function generator connect a 1v p–p sine wave at 100KHz/1 MHz/or 10 MHz to the Keysight N9914A spectrum analyzer (SA)/network analyzer (NA) input as shown in Figure 17.

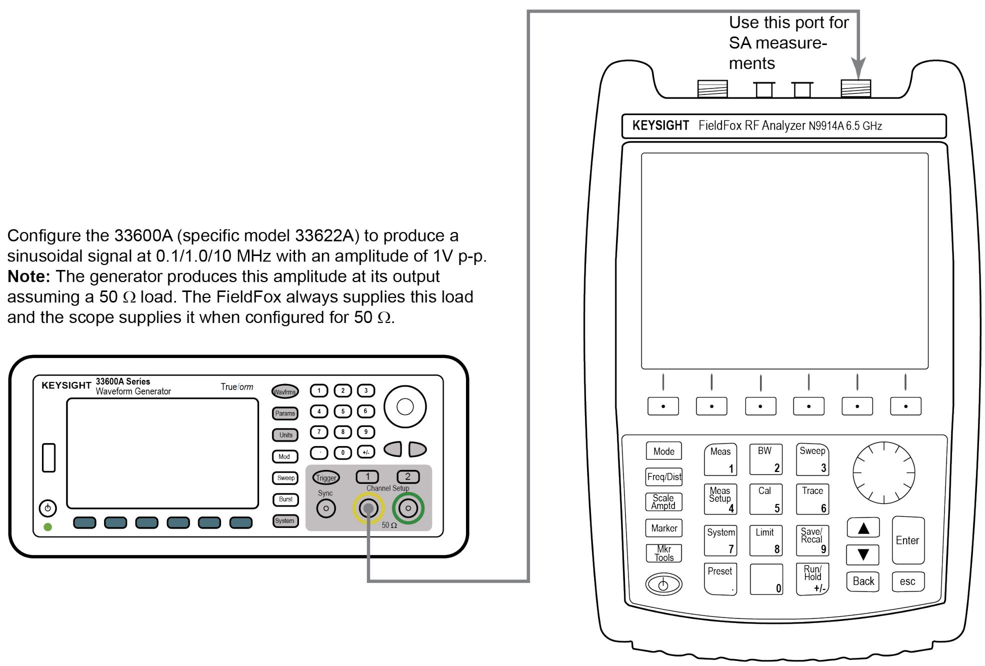


Figure 17: Set–up for basic spectrum analysis using the N9914A and 33600A.

The 1v p–p amplitude can be set using the Agilent MSO–X–6004A oscilloscope provided the scope input impedance set for 50Ω input impedance. The measurements you will be taking are with respect to a 50 ohm impedance system. In a linear systems theory course a 1 ohm impedance environment is often assumed. This is convenient for mathematical modeling purposes, but in practice 50 ohms (75 ohms for cable TV) is the way radio frequency (RF) measurements are actually taken. The most common display mode for a spectrum analyzer is power in dB relative to one milliwatt. The units are denoted dBm. To find the power in dBm delivered to a 50 ohm load, from a single sinusoid waveform, we might start with a scope waveform reading of volts peak–to–peak or peak. From basic circuit theory, the power delivered to a load R is

$$P_l = \frac{1}{2} \cdot \frac{v_{\text{peak}}^2}{R} = \frac{1}{2} \cdot \frac{(v_{\text{pp}}/2)^2}{R} = \frac{v_{\text{rms}}^2}{R} \text{ watts.} \tag{37}$$

Recall that for a sinusoid, the rms value is
$v_{\mathrm{peak}}/\sqrt{2}$, and clearly the peak value is one half the peak–to–peak value. In dBm the power is

$$(P_l)_{\mathrm{dBm}} = 10\log_{10}\left[\frac{P_l\,(\mathrm{watts})\cdot 1000\ \mathrm{mW/W}}{1\ \mathrm{mW}}\right] = 10\log_{10}\left[P_R\,(\mathrm{watts})\right] + 30 \qquad (38)$$

Suppose we have a 1v rms sinusoid in a 50 ohm measurement environment (that is 1 V rms across the load imposed by the analyzer). The power level is

$$(P_l)_{\mathrm{dBm}} = 10\log_{10}\left(1^2/50\right) + 30 = 13.01\ \mathrm{dBm} \qquad (39)$$

For the problem at hand, measure the input signal amplitude directly from the spectrum analyzer cursor by positioning the *Marker* at the sinusoid frequency.

Problem 2

By observing an oscilloscope display, adjust the output of a function generator to obtain a 1v p–p square wave at 100KHz. Connect this signal to the spectrum analyzer. Note that you can *sharpen up* the frequency spike for each of the harmonics if you reduce the resolution bandwidth setting. On the N9914A use the BW button (#2) to manually set the Resolution (res) bandwidth to say 1 kHz. Ask your lab instructor for assistance if you are having problems.

Part a

Identify and read the amplitudes of the various harmonics contained in the input signal $x(t)$. Compare your measured values with theoretical calculations obtained from Fourier analysis.

Part b

Investigate the harmonic content of a triangle waveform, again compare the experimental results with theoretical calculations. Note the Keysight 33600 upper frequency limit for triangle waves is just 800 kHz, so if you were using a fundamental frequency of 1 MHz or 10 MHz you should switch to 100 kHz for this part of the lab.

Problem 3

Generate a square wave at 10 MHz to produce a total output power of –20 dBm. Observe this signal on the spectrum analyzer and configure the display to start at 5 MHz and stop at 55 MHz. Measure/observe the power levels in dBm of the 1st, 3rd, and 5th harmonics. Using Fourier series theory predict the power level of the first through third harmonics you expect to measure given that the total power is –20 dBm.

Problem 4

In this problem you will take network analyzer measurement of three filters found on the RB Board. Two filters are bandpass and one is lowpass. The test configuration when using the network analyzer (NA) capability of the FieldFox is shown in Figure 18.

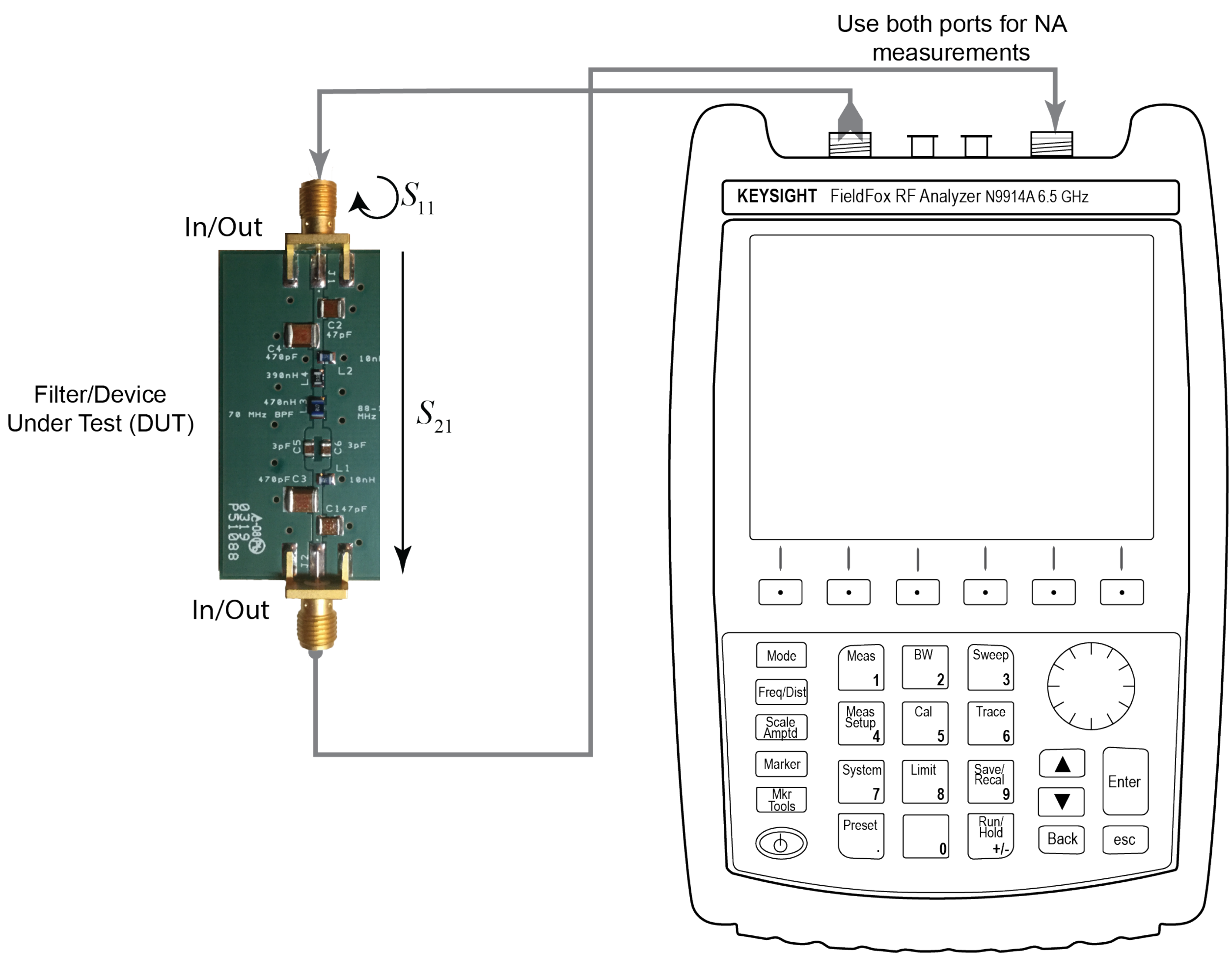


Figure 18: Test setup for measuring the S–parameters of a filter using the N9914A.

Filter Design Requirements

The design specifications of the three filters is given Table 17.

Table 17: Filter design specifications

Filter Type	Description
70 MHz BPF	$N = 3$ lowpass prototype, with 0.5 dB passband ripple and 0.5 dB bandwidth of 10 MHz centered on 70 MHz. The fractional bandwidth is thus $w = \mathrm{BW}/f_0 = 10/70 = 14.28\%$, where $\mathrm{BW} = f_2 - f_1$ and $f_0 = \sqrt{f_1 f_2}$.
88–108 MHz BPF	$N = 3$ lowpass prototype, with 0.5 dB passband ripple, lower band edge of 87 MHz, upper band edge of 109 MHz. The fractional bandwidth is thus $w = (109 - 87)/\sqrt{87 \times 109} = 22.6\%$.
1 MHz LPF	$N = 5$ lowpass prototype, with 0.5 dB passband ripple and cutoff frequency of 1 MHz.

Theoretical frequency response plots of these filters can be found in the Lab 1 sample notebook. The board schematics are given In Figures 19 and 20.

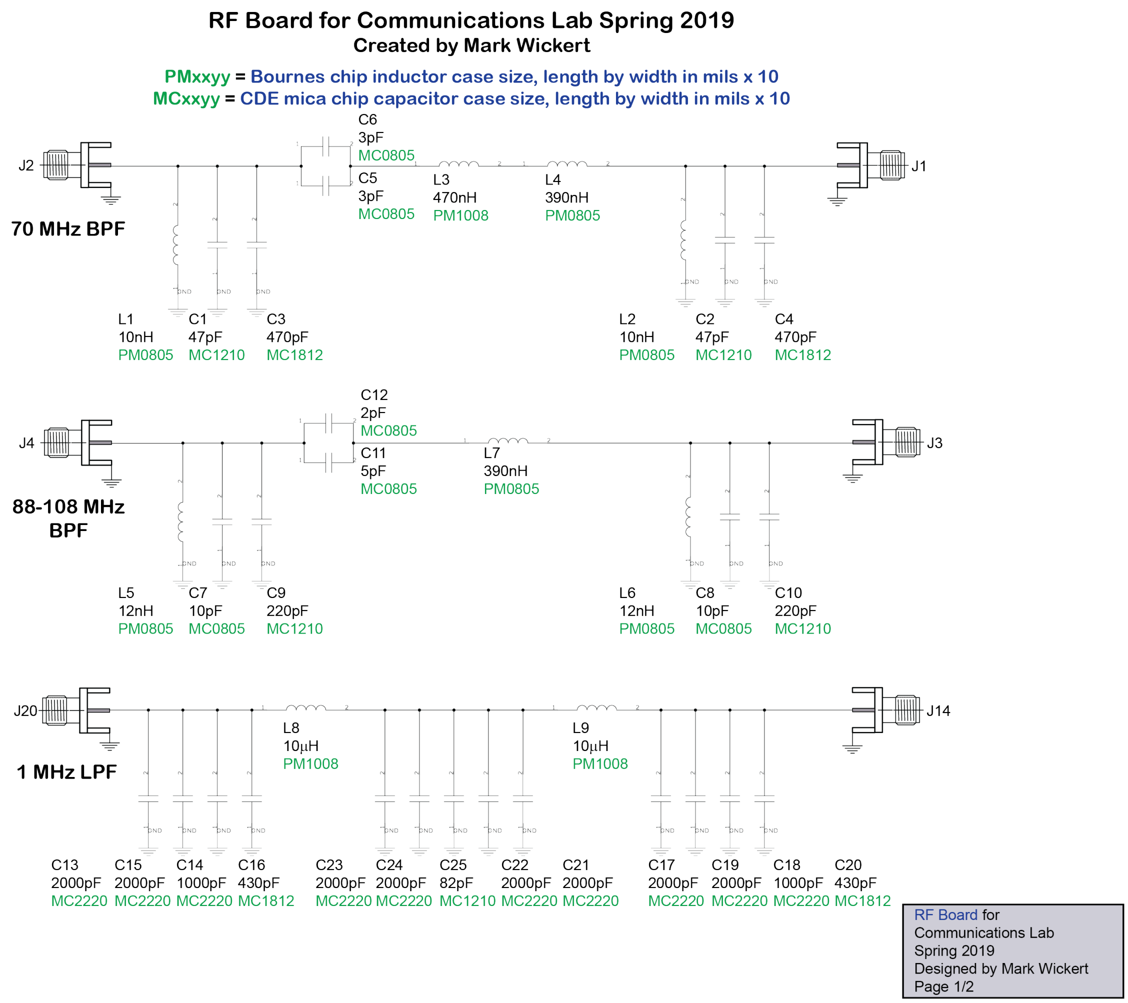


Figure 19: RF board schematic page one showing the passive LC filter designs.

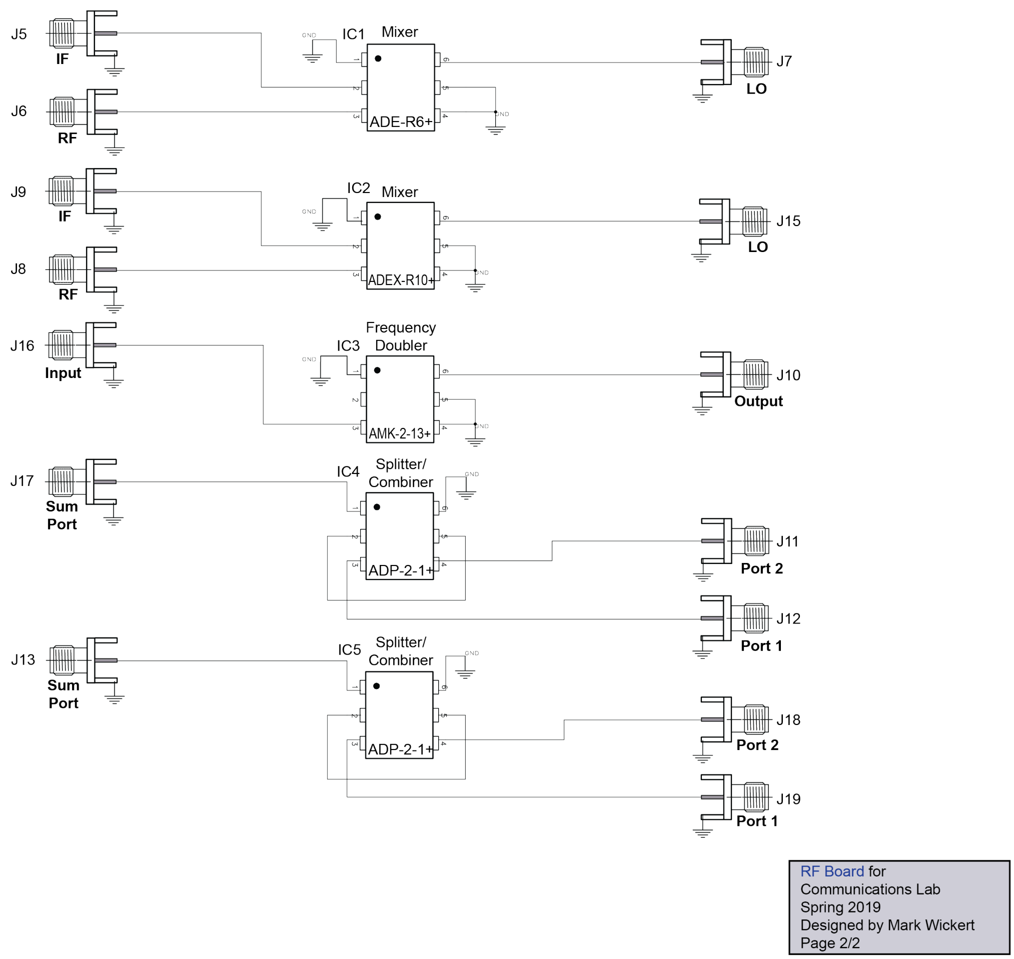


Figure 20: RF board schematic page two showing the Mini Circuits RF components, used in later labs.

Modeling Implementation Impairments

A Jupyter notebook supplies the needed math modeling tools that were used to first design the filters and now can be used to understand performance limitations of the filters. These limitations include:

- Component tolerances, in particular the inductors which presently have 20% tolerance; the capacitor tolerance is only 5%
- Losses due to the inductor finite quality factor Q, which introduce a frequency dependent series resistance for series resonators and a shunt resistance for parallel resonators

Using the Jupyter notebook you can determine component values. For example the notebook shows values for an $N = 3$ $f_0 = 30$ MHz design and then how to take measured values and place them in a notebook by reading `*.s2p` files exported from the FieldFox. Once in the notebook overlay plots with loss and component tolerance impairments can be used to compare to what you have measured.

The slider interface example is shown in Figure 21. The Jupyter notebook code is listed below.

```
p_L1 = widgets.FloatSlider(value=0.,min=-20,max=20,step=0.1,description='L1 tol in %')
p_C1 = widgets.FloatSlider(value=0.,min=-5,max=5,step=0.1,description='C1 tol in %')
p_Q1 = widgets.FloatLogSlider(value=1000.,base=10,min=1,max=3,step=0.02,description='Q1')
p_L2 = widgets.FloatSlider(value=0.,min=-10,max=10,step=0.1,description='L2 tol in %')
p_C2 = widgets.FloatSlider(value=0.,min=-5,max=5,step=0.1,description='C2 tol in %')
p_Q2 = widgets.FloatLogSlider(value=1000.,base=10,min=1,max=3,step=0.02,description='Q2')
p_L3 = widgets.FloatSlider(value=0.,min=-20,max=20,step=0.1,description='L3 tol in %')
p_C3 = widgets.FloatSlider(value=0.,min=-5,max=5,step=0.1,description='C3 tol in %')
p_Q3 = widgets.FloatLogSlider(value=1000.,base=10,min=1,max=3,step=0.02,description='Q2')
uiV1 = widgets.VBox([p_L1,p_C1,p_Q1])
uiV2 = widgets.VBox([p_L2,p_C2,p_Q2])
uiV3 = widgets.VBox([p_L3,p_C3,p_Q3])
ui = widgets.HBox([uiV1,uiV2,uiV3])

def f_BPF(p_L1,p_C1,p_Q1,p_L2,p_C2,p_Q2,p_L3,p_C3,p_Q3):
    # Bring some variables into this function as globals to make the input arg
    # list shorter; other approaches work too, but this was an easy fix
    global L_BPF30, C_BPF30, f, S21magdB
    #f = arange(20,40,.01)*1e6
    # Tune the L and C values away from ideal and
    L_tol_err = array([(1+p_L1/100)*L_BPF30[0],(1+p_L2/100)*L_BPF30[1],(1+p_L3/100)*L_BPF30[2]])
    C_tol_err = array([(1+p_C1/100)*C_BPF30[0],(1+p_C2/100)*C_BPF30[1],(1+p_C3/100)*C_BPF30[2]])
    H_ideal = lc6_Ymodel_bpf(f,L_BPF30,C_BPF30,30e6,Q=[1000,1000,1000])
    H_loss = lc6_Ymodel_bpf(f,L_tol_err,C_tol_err,30e6,Q=[p_Q1,p_Q2,p_Q3])
    plot(f/1e6,20*log10(abs(H_ideal)))
    plot(f/1e6,20*log10(abs(H_loss)))
    plot(f/1e6,S21magdB)
    #semilogx(f,20*log10(abs(H)))
    ylabel(r'Gain (dB)')
```

```
    xlabel(r'Frequency (MHz)')
    title(r'30 MHz $N=3$ BPF')
    legend((r'Ideal Model',r'Loss/Tol. Model',r'Measured'))
    grid();

output = widgets.interactive_output(f_BPF,{'p_L1': p_L1, 'p_C1': p_C1, 'p_Q1': p_Q1,
                                           'p_L2': p_L2, 'p_C2': p_C2, 'p_Q2': p_Q2,
                                           'p_L3': p_L3, 'p_C3': p_C3, 'p_Q3': p_Q3})
display(ui, output)
```

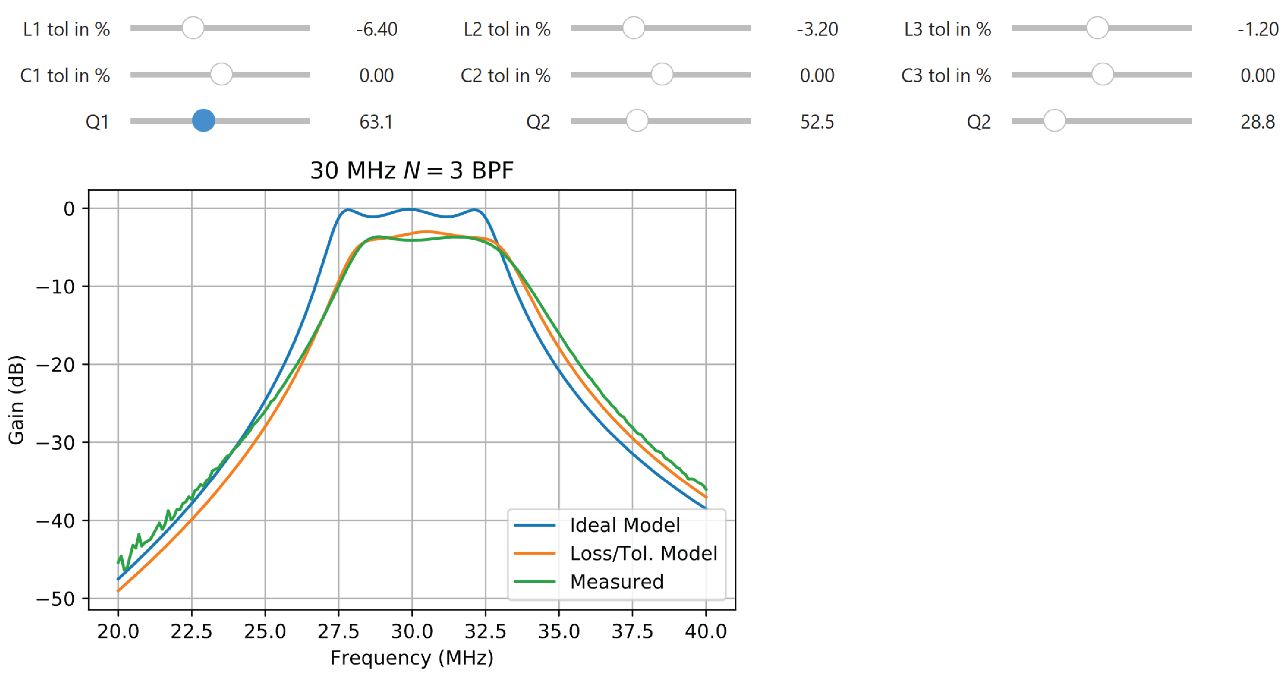


Figure 21: Jupyter interface for studying tolerance and loss in passive LC filters.

To complete Problem 4 you should have a screen shot similar to Figure 21 for each of the three filters on the RF Board. I say *screenshot* since this will be the easiest way to capture the state of the slider controls that gives the best fit for your report and includes the corresponding plot.

Problem 5

In this final problem you will experiment with the design (not implementation) of a $f_0 = 70$ MHz BPF similar to the one on the board, but now the bandwidth is reduced from 10 MHz to 4 MHz. I want you to see how challenging it is to design on paper this filter using standard chip capacitor and chip inductor values. In this exercise only consider ideal components and commercial off-the-shelf (COTS) component values, including simple series parallel combinations.

Design the filter and plot the theoretical response as shown in the sample Jupyter notebook. Now select parts from the provided parts catalog to try to get close to the theory values. Overlay the response of your COTS design on top of the ideal theory design.

The inductor-part1 inductor-part2 and capacitor catalogs are linked from the course Web Site.

References

1. Rodger Ziemer and William Tranter, *Principles of Communications*, 8th edition, Wiley, 2014.
2. David M. Pozar, *Microwave Engineering*, fourth edition, John Wiley, New York, 2011.
3. Robert E. Collin, *Foundations for Microwave Engineering*, second edition, McGraw-Hill, New York , 1992.
4. https://rf-tools.com/lc-filter/.
5. https://www.electronics-tutorials.ws/accircuits/parallel-resonance.html.
6. https://commonmark.org/.

Appendix

TBD