ECE 4670 Spring 2014 Lab 5
Frequency Modulation, Demodulation, and Phase-Locked Loops

1 Introduction

The underlying theme of this lab is angle modulation, in particular frequency modulation (FM). The details of both modulation and demodulation are investigated. Particular emphasis will be placed on the modulation process using a voltage controlled oscillator. The spectrum of an FM signal will also be examined. The use of the quadrature detector and/or the phase-locked loop for FM demodulation is also considered.

1.1 FM Frequency Deviation Constant

When the instantaneous frequency of a sinusoidal carrier waveform is proportional to a message, $m(t)$, it can be expressed as

$$f_i = f_c + f_d\, m(t) \tag{1}$$

where f_c is the carrier frequency, $m(t)$ is the modulating signal, and f_d is the frequency deviation constant with units of Hz/volt.

Since frequency is the time derivative of phase, or instantaneous phase is the integral of instantaneous frequency, the FM waveform can be expressed as

$$\begin{aligned} x_c(t) &= A_c \cos[\theta_c(t)] \\ &= A_c \cos\left[2\pi\left(f_c t + f_d \int^t m(\tau)\, d\tau\right)\right] \end{aligned} \tag{2}$$

When $m(t) = A_{dc}$ =a constant, the instantaneous frequency becomes

$$\begin{aligned} f_i &= \frac{d}{dt}[f_c t + f_d\, A_{\text{dc}} t] \\ &= f_c + f_d\, A_{\text{dc}} \end{aligned} \tag{3}$$

That is, a dc voltage produces a frequency that is offset from the carrier frequency by $f_d\, A_{\text{dc}}$ Hz.

1.1.1 Laboratory Exercises

Using a power supply, dc voltmeter, and frequency counter, measure the frequency versus dc voltage input for the Agilent 33250A function generator. Of the two synthesized sources, only the Agilent 33250A can can be used as a voltage controlled oscillator (VCO). As shown in Figure 1, the 33250A is first set for FM modulation, and then an external modulation source, applied via a rear panel connector, is selected. On the front panel select a deviation of 100 kHz. When in external FM mode the deviation value is no longer the peak deviation. Instead it is the peak deviation about the carrier when the modulation input swings to ±5v. So for an input of +5 v the generator output should increase in frequency by 100 kHz. For an input of -1 v the generator output should decrease by $100/5 \times 1 = 20$ kHz. Reference all data that you take in the lab to the rear panel input, hereafter referred to as v_{mod}.

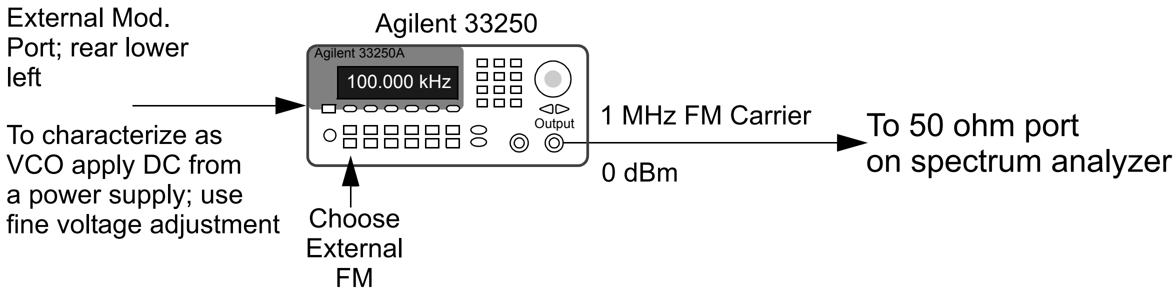

Figure 1: Configuring the Agilent 33250A as a VCO

1. Enter the frequency versus voltage data you collect into MATLAB for plotting. The applied voltage range of interest is $-5 < A_{\text{dc}} < 5$ volts as applied at the v_{mod} input with the carrier center frequency is set to 200 kHz and the front panel deviation is 100 kHz. Take data points about 0.5 volts apart as long as your graph is linear. Clearly establish the region of linearity of control by taking more points near the ends as required. With the data in MATLAB you can easily *fit* a line to the data using *Basic Fitting*. First create a plot of the frequency versus voltage data using

   ```
   >> plot(x_voltage, y_frequency, '.')
   ```

   This plot will be just the points with no line connecting them. Next select the plot figure tab and then go to the Tools menu and choose Basic Fitting. From the floating window click Linear, click Show Equations and increase the significant digits to four. Now you will have a nice *least squares* linear fit to your data with the slope of the curve fit available for use in further calculations, forthcoming.

2. Repeat the above for carrier center frequency settings of 1 MHz and 10 MHz, keeping the front panel deviation value set to 100 kHz.

3. Compute f_d for the Agilent 33250A with respect to the v_{mod} input using the data you obtained above for each center frequency. Note that according to Agilent, in all cases we expect $f_d = 100/5 = 20$ kHz/v.

1.2 FM Modulation Index - β

If we let $m(t) = A_m\, sin[2\pi f_m t]$ in the earlier equation given for $x_c(t)$, then

$$\begin{aligned} x_c(t) &= A_c\, \cos\left[2\pi f_c t + \frac{A_m f_d}{f_m}\cos(2\pi f_m t)\right] \\ &= A_c\, \cos\left[2\pi f_c t + \beta\, \cos(2\pi f_m t)\right] \end{aligned} \tag{4}$$

In the above equation, $A_m\, f_d/f_m = \beta$ is referred to as the modulation index. It is an important parameter for characterizing the FM signal. Note that β increases with increasing amplitude of the modulating signal, and decreases with increasing frequency, f_m.

The equations given here represent sinusoidal frequency modulation, or tone modulation with frequency f_m. The equation is periodic and has a Fourier series. The series is rather complicated, however, and has coefficients which are functions of β instead of being fixed constants as in the case, for example, of a square wave. The Fourier series representation of sinusoidal frequency modulation is given by

$$\begin{aligned} x_c(t) = {} & A_c J_0(\beta) \cos \omega_c t \\ & + A_c J_1(\beta)[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t] \\ & + A_c J_2(\beta)[\cos(\omega_c + 2\omega_m)t + \cos(\omega_c - 2\omega_m)t] \\ & + A_c J_3(\beta)[\cos(\omega_c + 3\omega_m)t - \cos(\omega_c - 3\omega_m)t] \ \ldots \end{aligned} \tag{5}$$

This equation shows that for tone modulation the spectrum consists, theoretically at least, of sidebands spaced f_m apart out to infinite frequency. The function $J_n(\beta)$ is called the n-th order Bessel function, of the first kind, with argument β. The Bessel functions are graphed and tabulated in most math handbooks, and the value of the function for any given argument, β, can easily be found.

For $\beta = 0,\ J_0(\beta) = 1$ and all of the remaining coefficients are zero. This says that the only term present in the series in the unmodulated case is the carrier frequency, as it should be. For β very small the first pair of sidebands will come and go according to the value of their corresponding coefficients in the equation above.

1.2.1 Laboratory Exercises

1. Connect the output of the Agilent 33250A function generator to a spectrum analyzer. Use a carrier frequency of about 1 MHz and center the carrier signal on the analyzer screen. Set the frequency span to about 10 kHz per division.

2. Connect a sinusoidal modulating signal (Agilent 33120A) to the v_{mod} input of the 33250A with a frequency of 10 kHz. Start with the modulating signal at zero amplitude and **slowly** increase the level. Watch the 33250A output on a scope to observe the frequency modulated waveform and also observe the frequency spectrum. As the input level is increased, one pair of sidebands and then a second and a third pair will appear. Reduce the input until only the first pair is present. On a dB scale we will call this the 10% point or the when the second pair of sidebands is down 20dB relative to the first pair. Take data to calculate the value of β at this point. The condition where only one sideband of the modulating frequencies is present is known as *narrow-band* FM. The maximum modulation index for sinusoidal narrow-band FM is usually assumed to be around $\beta = 0.5$ or less. Would you agree?

3. Increase the modulation amplitude slowly, observing the FM waveform and noting the appearance of several additional sidebands on the analyzer screen. Use the 10 dB per division vertical scale. At some point, after three or four sidebands have appeared, the carrier frequency line will begin to decrease in amplitude. Adjust the modulation amplitude until the carrier term is gone. Calculate β for this condition and compare your experimental value with theory. Note that the zeros of the $J_n(\beta)$ Bessel functions are tabulated in mathematical

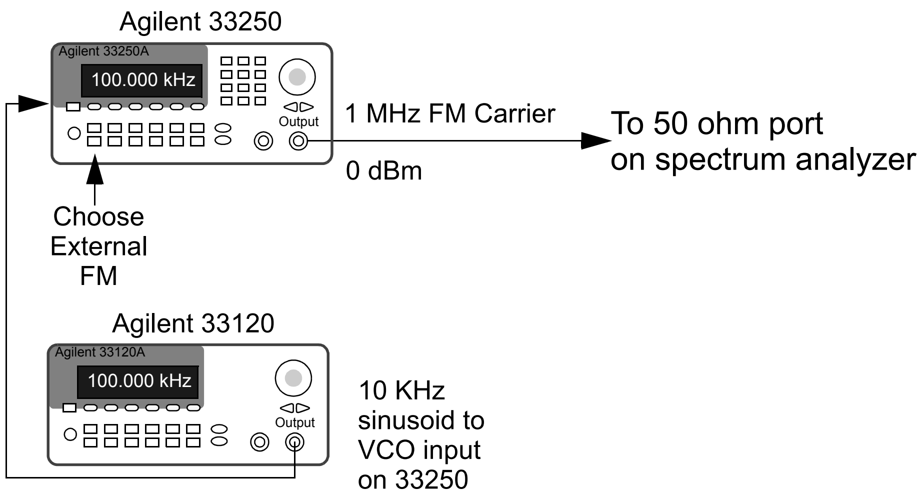

Figure 2: Sinusoidal FM using the combination of Agilent 33250A as a VCO and the Agilent 33120A as a modulation source.

handbooks and communication theory texts such as Ziemer and Tranter[1]. A short table of zeros is given below

	$J_n(x) = 0$				
$n = 0$	2.4048	5.5201	8.6537	11.7915	14.9309
$n = 1$	3.8317	7.0156	10.1735	13.3237	16.4706
$n = 2$	5.1356	8.4172	11.6198	14.7960	17.9598
$n = 3$	6.3802	9.7610	13.0152	16.2235	19.4094
$n = 4$	7.5883	11.0647	14.3725	17.6160	20.8269
$n = 5$	8.7715	12.3386	15.7002	18.9801	22.2178
$n = 6$	9.9361	13.5893	17.0038	20.3208	23.5861
$n = 7$	11.0864	14.8213	18.2876	21.6415	24.9349
$n = 8$	12.2251	16.0378	19.5545	22.9452	26.2668

4. Now use one-half the modulation frequency of that used above and, by adjusting the input signal amplitude, duplicate the conditions observed in 3. How do the amplitudes compare for these two cases? Does this agree with the equation for β?

5. Return to the frequency and amplitude of 3. Increase the input signal level slowly and note the signal amplitudes for which $J_1(\beta)$, $J_2(\beta)$, etc., go to zero. At what value of β does the carrier term go to zero a second time? Compare your results with theory. Additionally compare your results to simulation experiments run using the provided ADS files (`ece4670_Lab5_prj.zip`) and depicted in Figures 3–6.

6. The parameter β is sometimes written as $\beta = \Delta f / f_m$. That is, $f_d A_m = \Delta f$, and Δf is called the frequency deviation. It is the maximum instantaneous frequency or the peak

[1] R. Ziemer and W. Tranter, *Principles of Communications*, sixth edition, John Wiley, 2009, page 142.

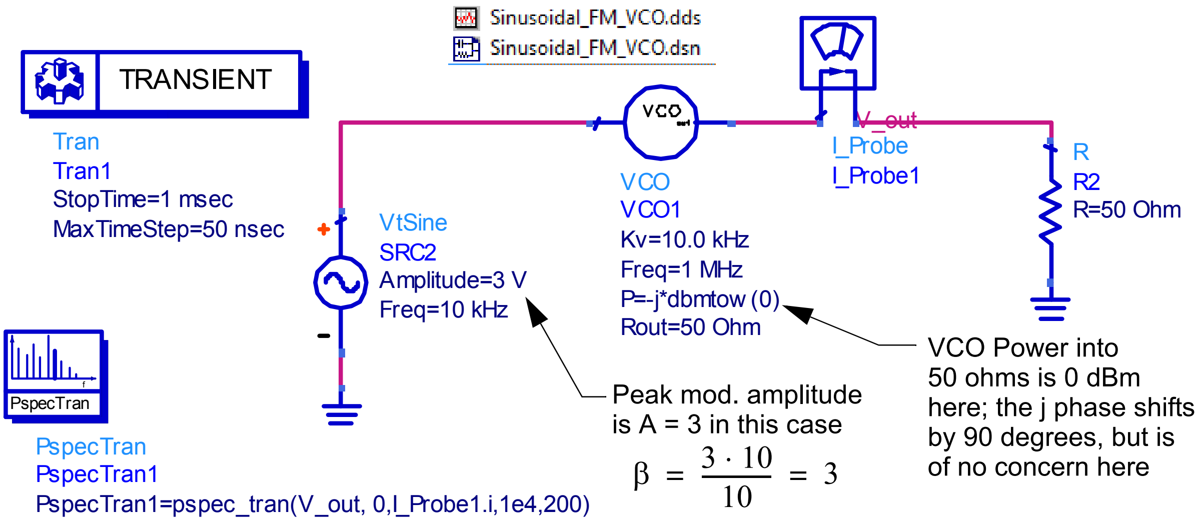

Figure 3: ADS simulation schematic.

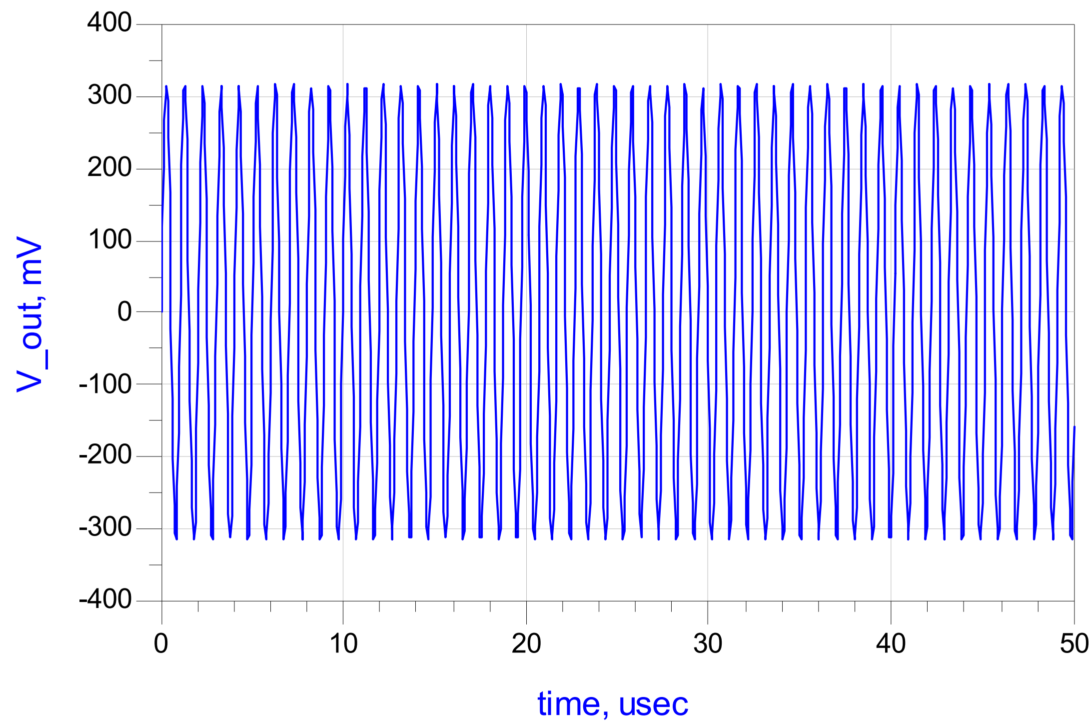

Figure 4: ADS FM output waveform.

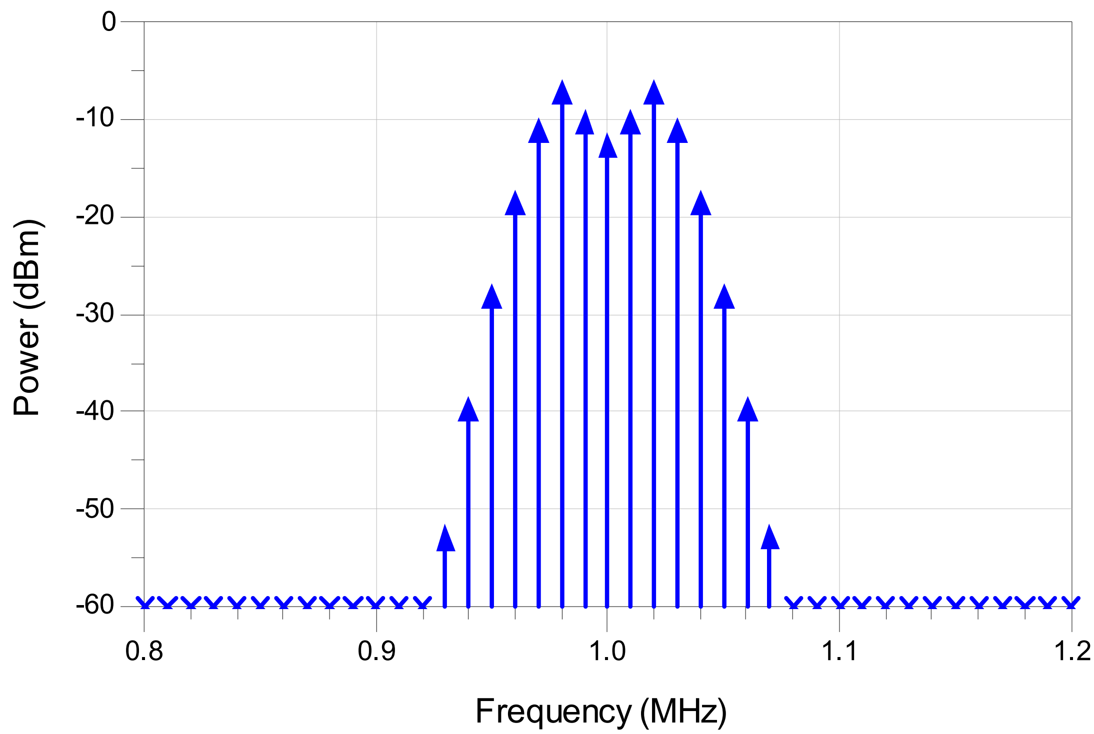

Figure 5: ADS FM spectrum with $\beta = 3$.

swing in carrier frequency from its unmodulated value. The bandwidth occupied by an FM signal is related to the peak deviation, but not in rigorous fashion. Set the peak deviation, $f_d A_m = \Delta f$, at a fixed value that gives sidebands out to about 100 kHz on each side of the carrier (use about 50 kHz per division on the analyzer).

7. Decrease the modulating frequency without changing Δf, and notice the effect on the spectrum. Measure the bandwidth at several different modulating frequencies. If you are using the 10dB per division vertical scale you can define spectral bandwidth in terms of the frequencies for which the spectrum is say 10 or 20dB down from its peak value. Calculate the relationship between Δf and bandwidth for each. Carson's rule for FM by a sinusoidal signal of frequency f_m states that the bandwidth is approximately

$$BW = 2 f_m(\beta + 1) = 2(\Delta f + f_m) \tag{6}$$

Would you agree with this approximation?

1.3 FM with Other than Sinusoidal Signals

The Fourier series (or transform) for an FM waveform is mathematically tractable for only a few special cases, the sinusoidal case being one of them. For signals which are more complicated, the detailed structure of the spectrum cannot be analyzed. Only a few rules relating modulating frequency, bandwidth, and peak deviation can be used to describe the frequency domain representation of FM for the general case.

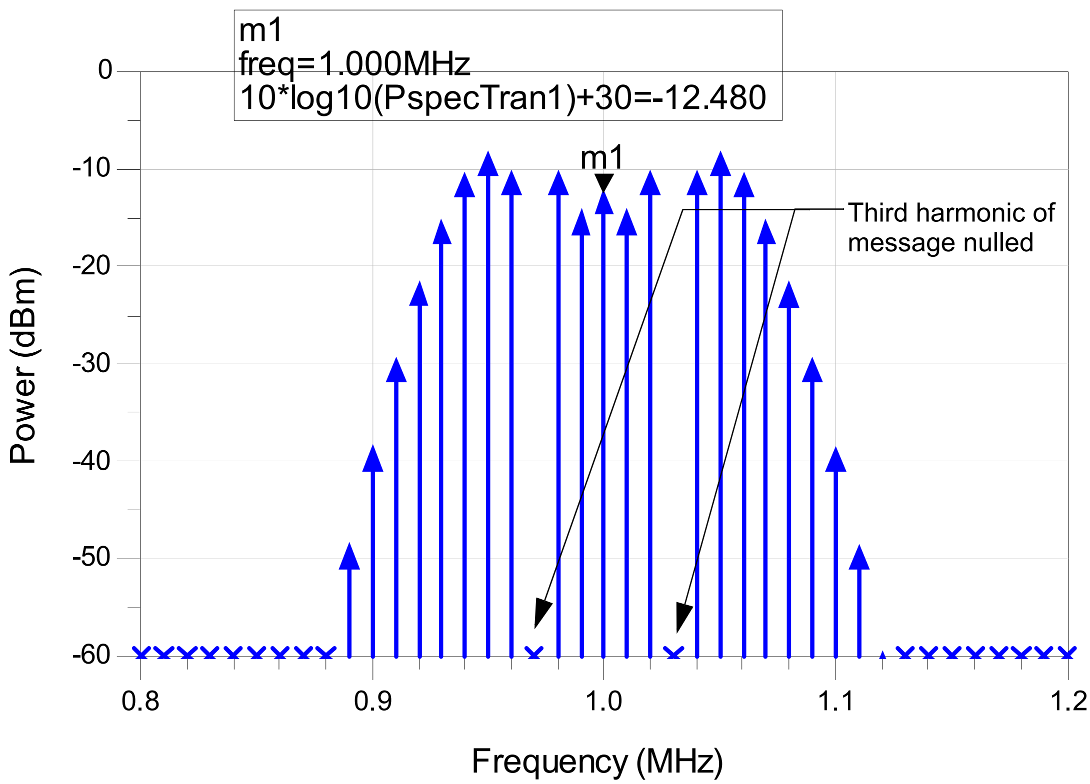

Figure 6: ADS spectrum with β such that the third sideband is nulled.

Suppose $m(t)$ is a zero average value square wave of amplitude $\pm A_m$. Then the instantaneous frequency of the resulting FM signal jumps from $(f_c - f_d A_m)$ to $(f_c + f_d A_m)$ or from $(f_c - \Delta f)$ to $(f_c + \Delta f)$. Mathematically we can write this as

$$x_c(t) = \sum_{n=-\infty}^{\infty} p(t - nT_m) \tag{7}$$

where

$$\begin{aligned} p(t) = A_c \Bigg\{ &\Pi\left(\frac{t - T_m/4}{T_m/2}\right)\cos[2\pi(f_c + \Delta f)t] \\ &+ \Pi\left(\frac{t - 3T_m/4}{T_m/2}\right)\cos[2\pi(f_c - \Delta f)t]\Bigg\} \end{aligned} \tag{8}$$

For this special case the power spectral density of $x_c(t)$ is relatively easy to obtain. Clearly $S_x(f)$ will consist of delta functions spaced at multiples of $1/T_m$. The envelope of $S_x(f)$ is proportional to $|P(f)|^2$.

1.3.1 Laboratory Exercises

1. Using the Agilent 33250A function generator with a carrier frequency of 1 MHz, apply a square wave signal of 10 kHz to the v_{mod} input. The test set-up is shown in Figure 7.

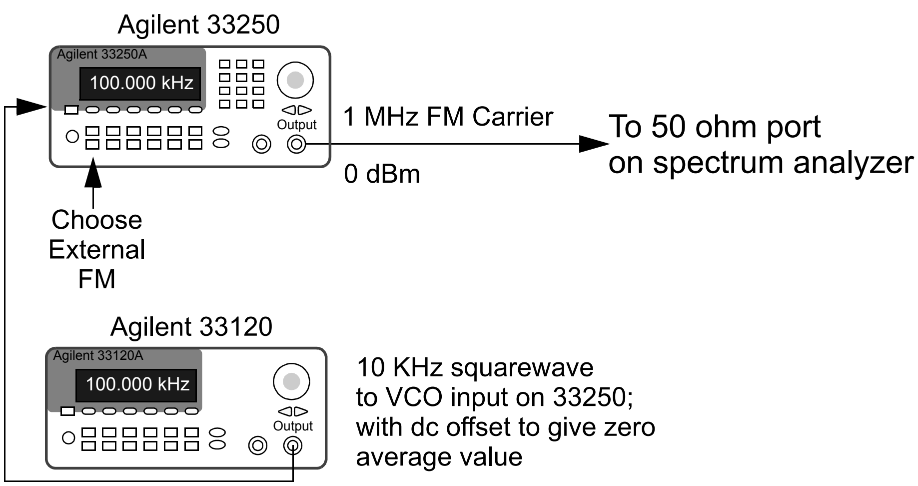

Figure 7: Squarewave FM using the combination of Agilent 33250A as a VCO and the Agilent 33120A as a modulation source.

2. Observe the modulated waveform on a scope triggered by the 10 kHz signal. Increase the frequency deviation of the Agilent 33250A function generator from 100 to 800 kHz. Try to adjust the modulating signal amplitude so that the two output frequencies are clearly evident. This should be observed at an input signal level of about 0.5v peak-to-peak, and at several more values up to as high as about 5.5v peak-to-peak. Try using a sweep rate of 20 μs/div on the oscilloscope.

3. Observe the spectrum on the analyzer as Δf is increased. Use the analyzer with zero frequency in the center and with 500 kHz per division so that both positive and "negative" frequencies can be observed. Compare your measured results to ADS simulation results similar that shown in Figures 8 and 9.

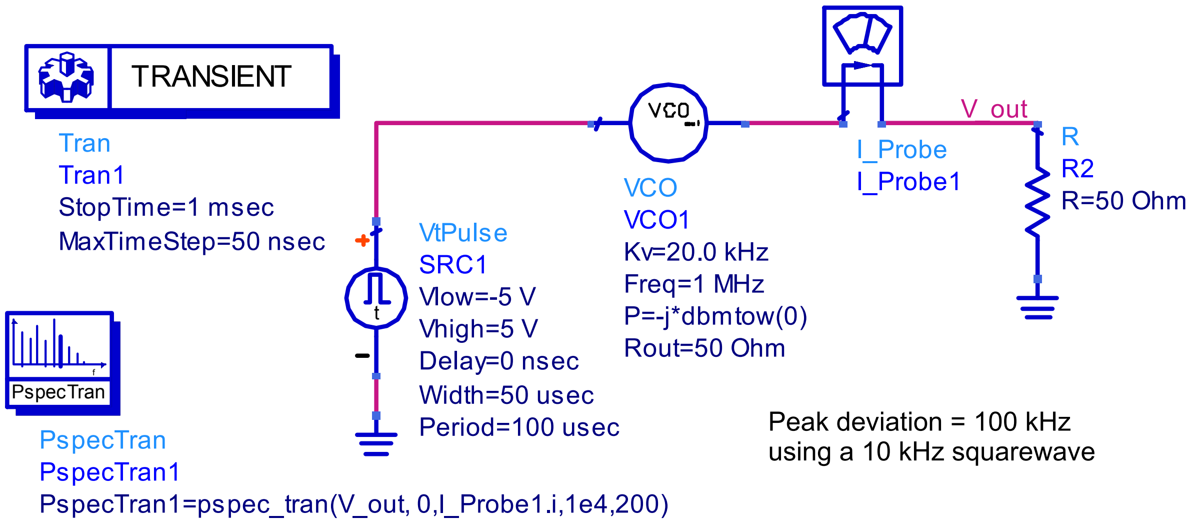

Figure 8: ADS simulation schematic for squarewave FM.

4. Slowly increase and decrease the modulation amplitude. Notice that at high levels the spectrum seems to cross the zero frequency line.

5. Return the 33250A frequency deviation back to 100 kHz as in Section 1.1.1. Set the amplitude for a ± 100 kHz bandwidth and decrease the modulation frequency without changing Δf. Does Carson's rule seem to hold for this signal?

6. Use an FM radio for a voice/music signal to modulate the 33250A generator. Set the bandwidth at about ± 100 kHz for this "random" signal and observe the spectrum. Manually increase the analyzer sweep speed so that you can more clearly see the constant changes in the spectral picture for this general case.

1.4 FM Demodulation

To recover the information contained in an FM signal requires obtaining the signals instantaneous frequency. For the signal

$$x_c(t) = A_c \cos[\omega_c t + \phi(t)] = A_c \cos\left[\omega_c t + k_f \int^t m(\alpha)\, d\alpha\right] \tag{9}$$

the instantaneous radian frequency is

$$\omega_i(t) = \omega_c + \frac{d\phi(t)}{dt} = \omega_c + k_f\, m(t) \tag{10}$$

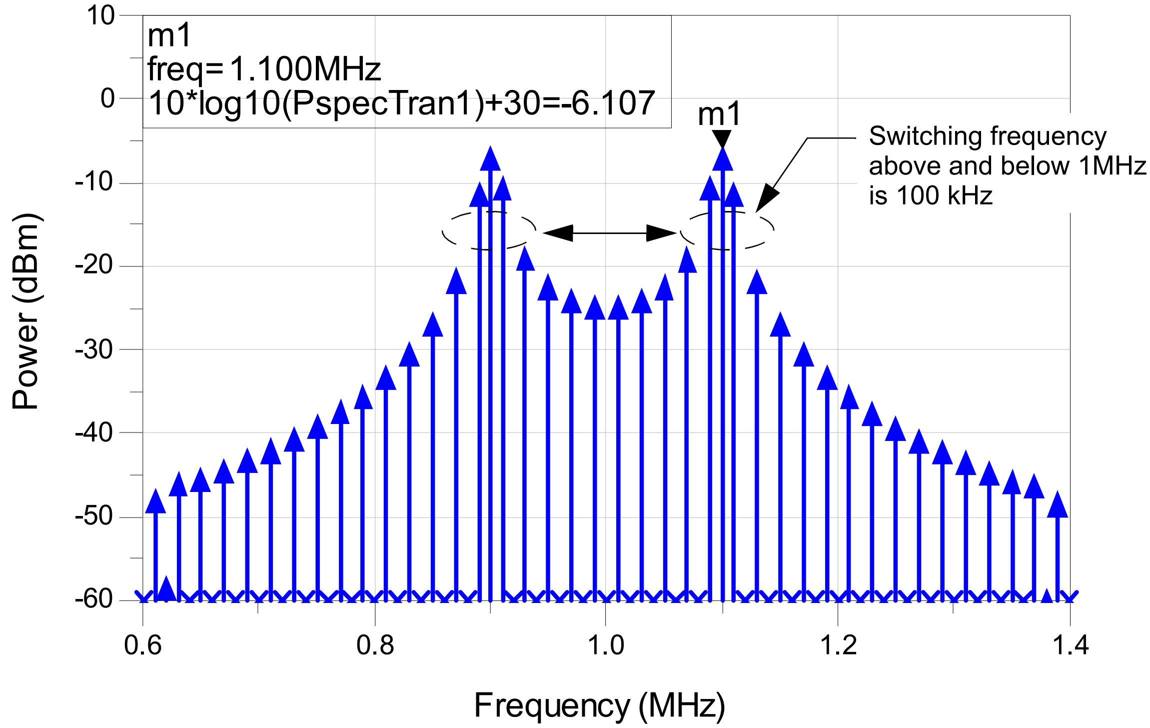

Figure 9: ADS squarewave FM spectrum with the peak deviation frequencies clearly visible.

Figure 10: Ideal FM discriminator

where $k_f = 2\pi f_d$. The ideal FM discriminator, shown in Figure 10, produces output

$$y_D(t) = \frac{1}{2\pi} K_D \frac{d\phi(t)}{dt} = K_D f_d m(t). \tag{11}$$

Practical implementation of the ideal FM discriminator can be done using analog circuit design or using digital signal processing. In this part of the lab we will consider the use of an analog phase-locked loop (PLL) with sinusoidal phase detector, as shown in Figure 11, for FM demodulation. For modeling purposes we let

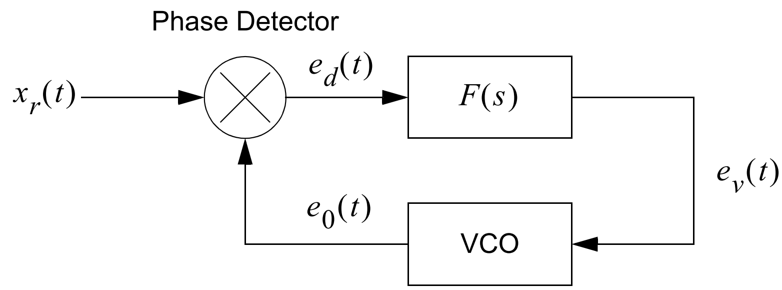

Figure 11: General PLL diagram employing a sinusoidal phase detector.

$$x_r(t) = A_c \sin\left[2\pi f_c t + \theta(t)\right] \tag{12}$$

$$e_o(t) = A_v \cos\left[2\pi f_c t + \hat{\theta}(t)\right]. \tag{13}$$

Note that frequency error may also be included in $\phi(t) = \theta(t) - \hat{\theta}(t)$. Assuming the double frequency term is removed, we can write

$$e_d(t) = \frac{1}{2} A_c A_v K_d \sin\left[\theta(t) - \hat{\theta}(t)\right]. \tag{14}$$

The VCO, see Figure 12, converts voltage to frequency deviation relative the VCO quiescent frequency f_0. The VCO output instantaneous frequency in Hz is

$$f_{\text{VCO}}(t) = f_0 + \frac{K_v}{2\pi} e_v(t) = f_0 + \frac{1}{2\pi} \cdot \frac{d\hat{\theta}(t)}{dt}. \tag{15}$$

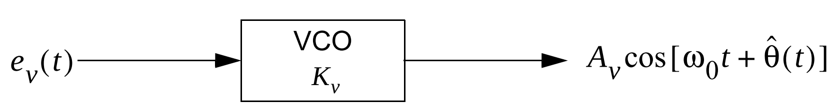

Figure 12: VCO model.

The frequency deviation in radians/s is

$$\text{VCO Frequency Deviation} = \frac{d\hat{\theta}(t)}{dt} = K_v e_v(t) \tag{16}$$

In mathematical terms we now close the loop by connecting the phase detector output to the VCO through a convolution of the loop filter impulse response

$$\frac{d\hat{\theta}(t)}{dt} = \frac{A_c A_v K_d K_v}{2} \int^t f(t-\lambda) \sin\left[\theta(\lambda) - \hat{\theta}(\lambda)\right] d\lambda \tag{17}$$

This equation can be represented in block diagram form as the nonlinear feedback control model of Figure 13.

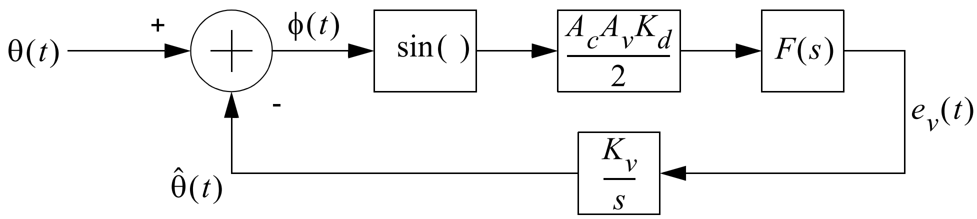

Figure 13: Non-linear baseband PLL model.

When the loop is in lock, with small phase error, i.e.

$$\sin[\phi(t)] = \sin[\theta(t) - \hat{\theta}(t)] \simeq \theta(t) - \hat{\theta}(t) = \phi(t), \tag{18}$$

we can linearize the loop. This linearizing leads to the s-domain PLL model shown in Figure 14. Working from the block diagram we can solve for $\Theta(s)$ in terms of $\Phi(s)$

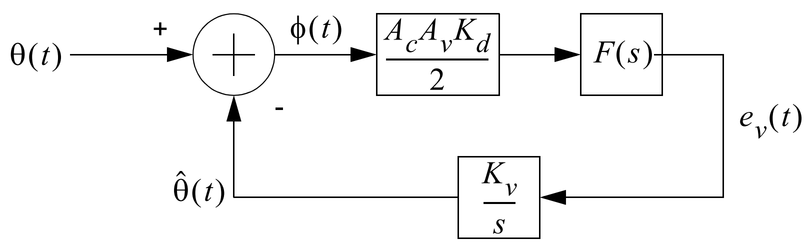

Figure 14: Linear baseband PLL model.

$$\begin{aligned} \hat{\Theta}(s) &= \frac{K_t}{s}\left[\Theta(s) - \hat{\Theta}(s)\right]F(s) \\ \text{or } \hat{\Theta}(s)\left[1 + \frac{K_t}{s}F(s)\right] &= \frac{K_t}{s}\Theta(s)F(s), \end{aligned} \tag{19}$$

where

$$K_t = \frac{1}{2}\mu A_c A_v K_d K_v) \text{ rad/s} \tag{20}$$

Finally, the closed-loop transfer function, $H(s) = \hat{\Theta}(s)/\Theta(s)$, can be written as

$$H(s) = \frac{\hat{\Theta}(s)}{\Theta(s)} = \frac{K_t F(s)}{s + K_t F(s)}. \tag{21}$$

For a first-order PLL $F(s) = 1$, then we have

$$H(s) = \frac{K_t}{s + K_t} \tag{22}$$

We are finally in a position to consider the details of how the first-order PLL recovers the FM message signal $m(t)$. With FM the phase deviation at the PLL input (from the FM transmitter) is

$$\Theta(s) = \frac{f_d M(s)}{s}, \tag{23}$$

where $M(s) = \mathcal{L}\{m(t)\}$. The VCO control voltage input is

$$\begin{aligned} E_v(s) &= \Theta(s) \cdot \frac{s}{K_v} \cdot H(s) \\ &= \frac{k_v M(s)}{s} \cdot \frac{s}{K_v} \cdot \frac{K_t}{s + K_t} \\ &= \frac{f_d}{K_v} \cdot \frac{K_t}{s + K_t} \cdot M(s). \end{aligned} \tag{24}$$

The 3dB bandwidth in Hz of the FM demodulator is just the loop gain divided by 2π

$$\text{Demodulator 3dB Bandwidth} = \frac{K_t}{2\pi} \text{ Hz} \tag{25}$$

The linear analysis assumes that the loop is in lock. The first-order PLL is in lock if $d\hat{\theta}(t)/dt = 0$. The governing relationship for the loop to be in lock is the nonlinear differential equation of (17). For the case of the first-order loop we have

$$\frac{d\hat{\theta}(t)}{dt} = K_t \sin[\phi(t)]. \tag{26}$$

Suppose the loop is in lock for $t < 0$ and the input phase deviation undergoes a step change in frequency, i.e.,

$$\frac{d\theta(t)}{dt} = \Delta\omega\, u(t), \tag{27}$$

where we assume $\Delta\omega > 0$. Combining this with (26), we can write

$$\frac{d\phi(t)}{dt} = \Delta\omega\, u(t) - K_t \sin[\phi(t)]. \tag{28}$$

A plot of $d\phi(t)/dt$ versus $\phi(t)$ is known as the *phase plane* plot. The phase plane plot for a first-order PLL having $\Delta\omega > 0$ is shown in Figure 15. At $t = 0$ the phase plane operating point jumps to $d\phi(t)/dt|_{t=0} = \Delta\omega$. Since dt is also positive, we conclude that $d\phi$ is also positive. If $d\phi(t)/dt$ should become negative $d\phi$ is also negative, which drives the operating point to the `stable lock point` which again has $d\phi(t)/dt = 0$ (a frequency error of zero). Due to the finite loop gain, K_t, there is a steady-state phase error ϕ_{ss} when the loop finally settles. We conclude that for the loop to lock, or in this case remain locked, the phase plane curve must cross the $d\phi/dt = 0$ axis.

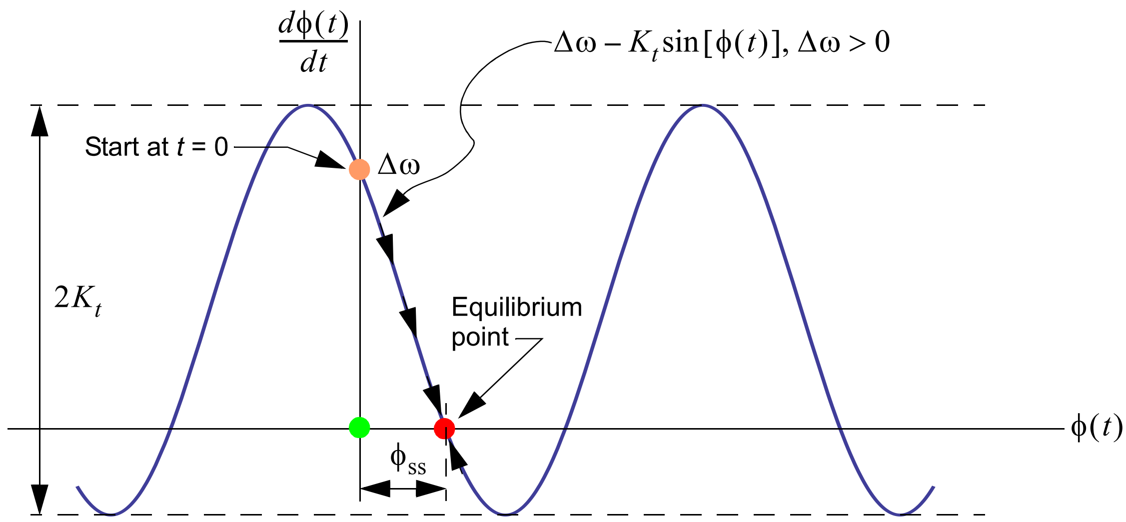

Figure 15: Phase plane plot for first-order PLL with a frequency step of $\Delta\omega > 0$.

The maximum $\Delta\omega$ the loop can handle is K_t rad/s, so the *total lock range* of the PLL is

$$\text{Total Lock Range in Hz: } f_0 - \frac{K_t}{2\pi} \le f \le f_0 + \frac{K_t}{2\pi}, \tag{29}$$

where we recall that f_0 is the VCO quiescent frequency. For a given $\Delta\omega$ within the lock range, the steady-state phase error is

$$\phi_{ss} = \sin^{-1}\left(\frac{\Delta\omega}{K_t}\right) \tag{30}$$

1.4.1 Laboratory Exercises

The PLL exercises focus on the test set-up of Figure 16, which uses the TFM-3 mixer board shown in Figure17. Note the SRA-3+ mixer of Lab 3 would also work, except the coupling capacitor on the IF port needs to be by bypassed.

1. Configure the Agilent 33250 (VCO) as shown in the Figure 16. Note that the loop filter in this case is just a wire from the phase detector directly to the VCO input (back panel of the 33250). Initially do not connect the phase output to the VCO input. Probe with the scope to see the difference frequency at the phase detector output. When the input signal and VCO are slightly offset in frequency you observe the difference frequency (*beat note*) on the scope. It is of the form

$$e_d(t) = K_m \sin(2\pi f_\Delta t), \tag{31}$$

which is actually of the same form of the phase detector output when the loop is locked, that is $K_m \sin(\phi)$. Note that the phase detector gain coefficient K_m is equivalent to $A_c A_v K_d/2$ shown in Figures 13 and 14. The phase detector gain, in volts per radian, is thus the peak voltage you observe on the scope. Record the value of K_m you observe for PLL closed-loop analysis.

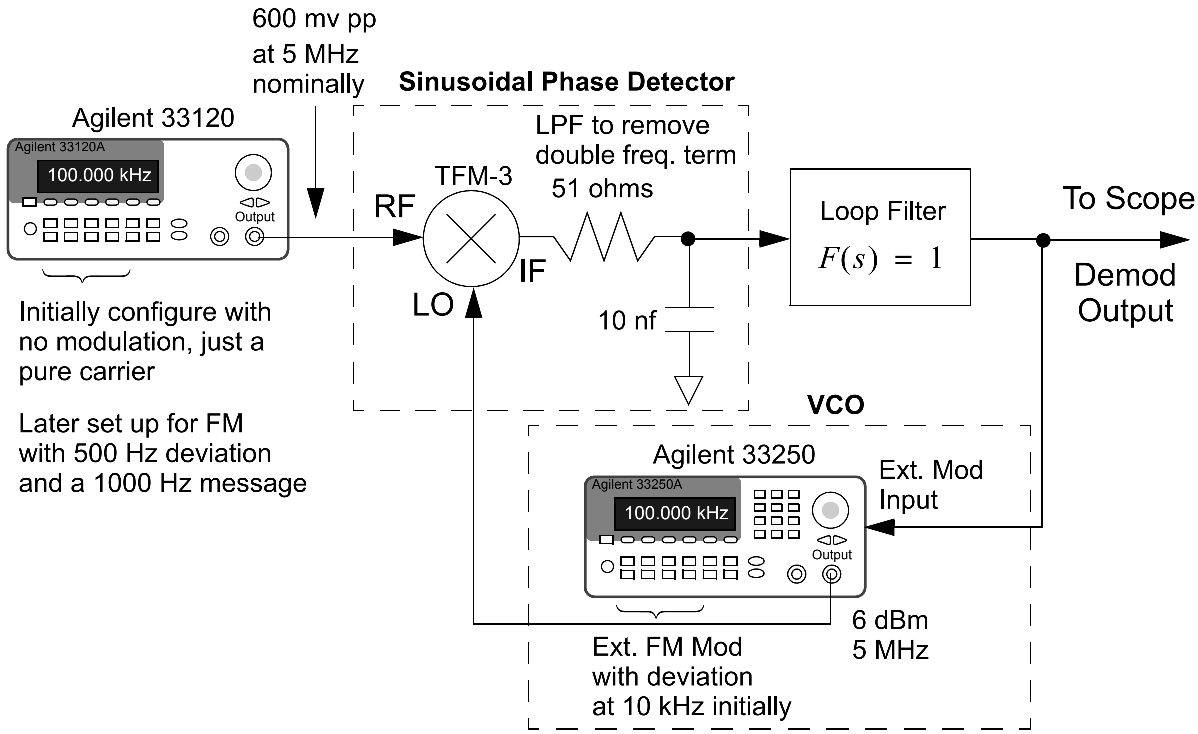

Figure 16: Instrument configuration for building a first-order PLL using a Minicircuits TFM-3 mixer as the phase detector.

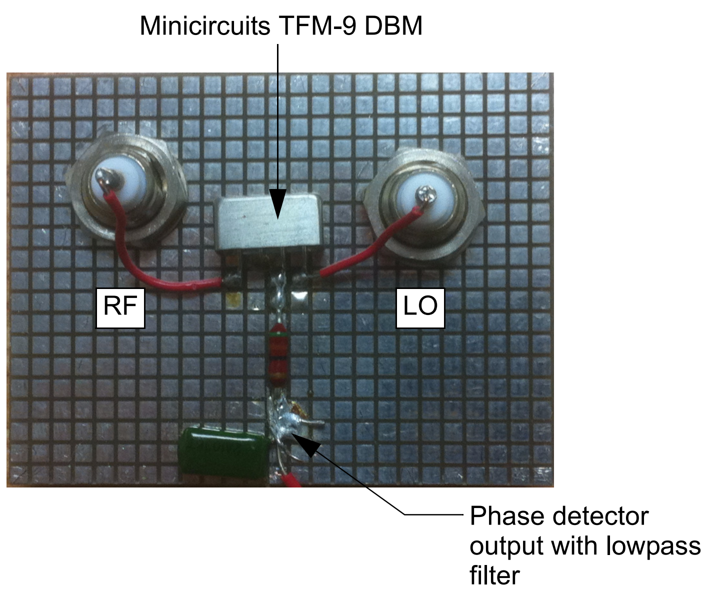

Figure 17: Minicircuits TFM-3 mixer/phase detector board.

2. Calculate K_t using the measured value of K_m and the previously measured value of the VCO sensitivity. Formally you need to have K_v in rads/s, but since we are most interested in the lock range in Hz, K_v in Hz/v is sufficient.

3. Now close the loop by connecting the VCO to the phase detector output. Verify that the loop is locked by observing the phase detector output on the scope using DC coupling. The beat note should be gone and you should see a DC level. You might need to lock the loop by tuning the input signal (Agilent 33120) in small 10 Hz steps above or below the nominal 5 MHz set value. Once the loop locks you will notice that the DC level you observe moves up and down with the frequency tuning of the input signal. Next measure the lock range of the PLL by varying the frequency of the input signal about 5 MHz. Take very small steps to insure that the loop does not jump out of lock before reaching the true upper and lower lock range limits. For the first-order PLL this should be twice the open loop gain in Hz, that is twice the product of the peak phase detector output voltage times the VCO sensitivity K_v in Hz/v. See if your calculations agree with your observation. The ADS simulation of Figures 18 and 18 shows what happens when the frequency of the input signal exceeds the lock range.

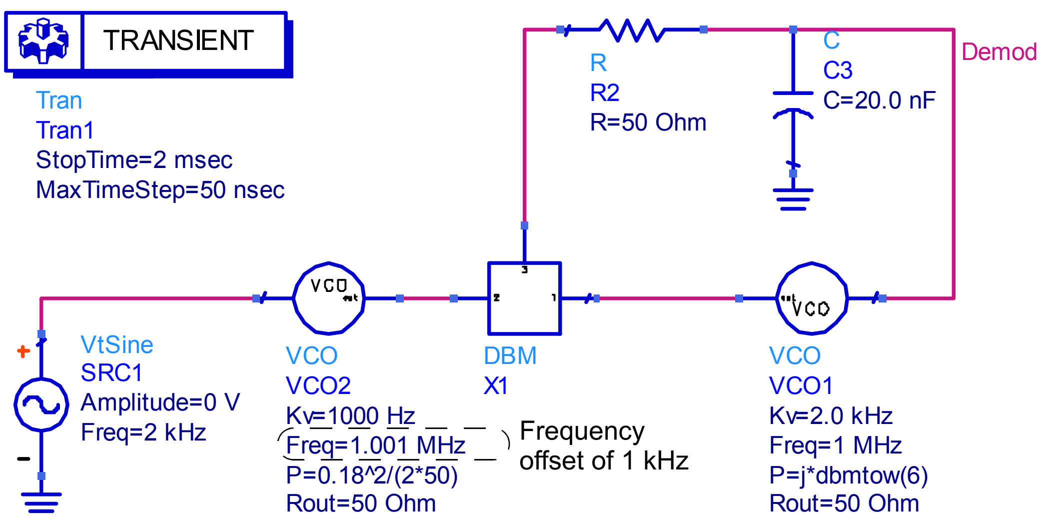

Figure 18: ADS first-order PLL model for lock range experimentation.

4. Now apply FM modulation by properly configuring the Agilent 33120 as shown in Figure 16. This generator is not quite as easy to set for FM as the Agilent 33250. Assuming the closed-loop bandwidth is wider than 1000 Hz, the PLL should be tracking the FM input signal, that is the VCO control voltage (phase detector output voltage) will follow the modulation. Verify this on the scope. This waveform is the demodulated FM signal. Compare your results with the ADS simulation shown in Figures 20 and 21.

5. Verify that if you increase the FM deviation of the input too far above 500 Hz the PLL will loose lock. Increase the gain of the PLL by increasing the deviation setting on the 33250

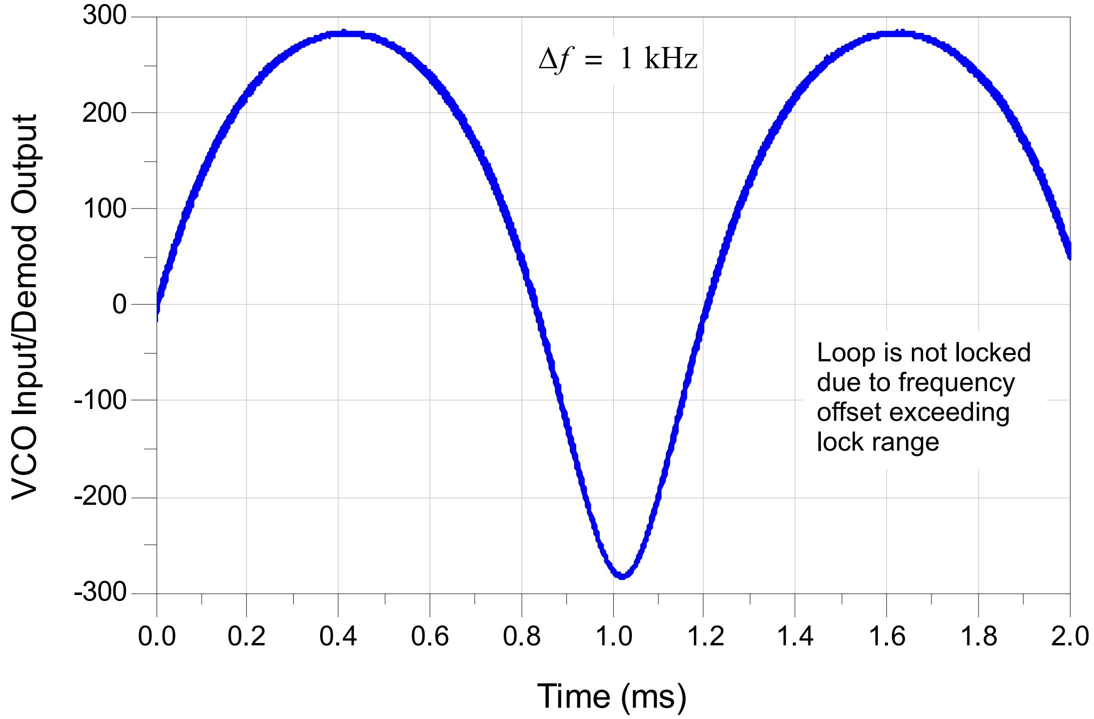

Figure 19: ADS first-order PLL phase detector output/VCO input when frequency offset exceeds the lock range (1 kHz > 563.2 Hz in this case).

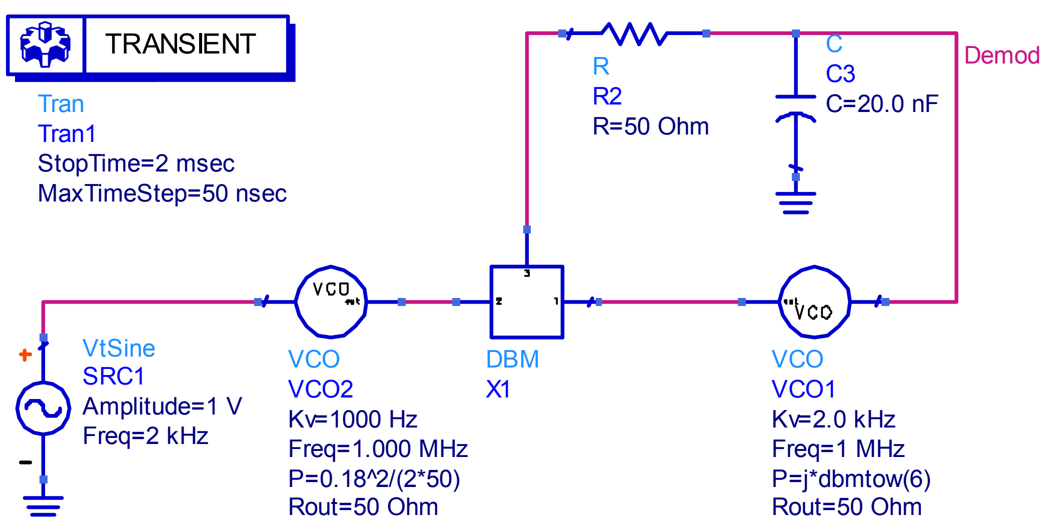

Figure 20: ADS first-order PLL model for demodulation of FM.

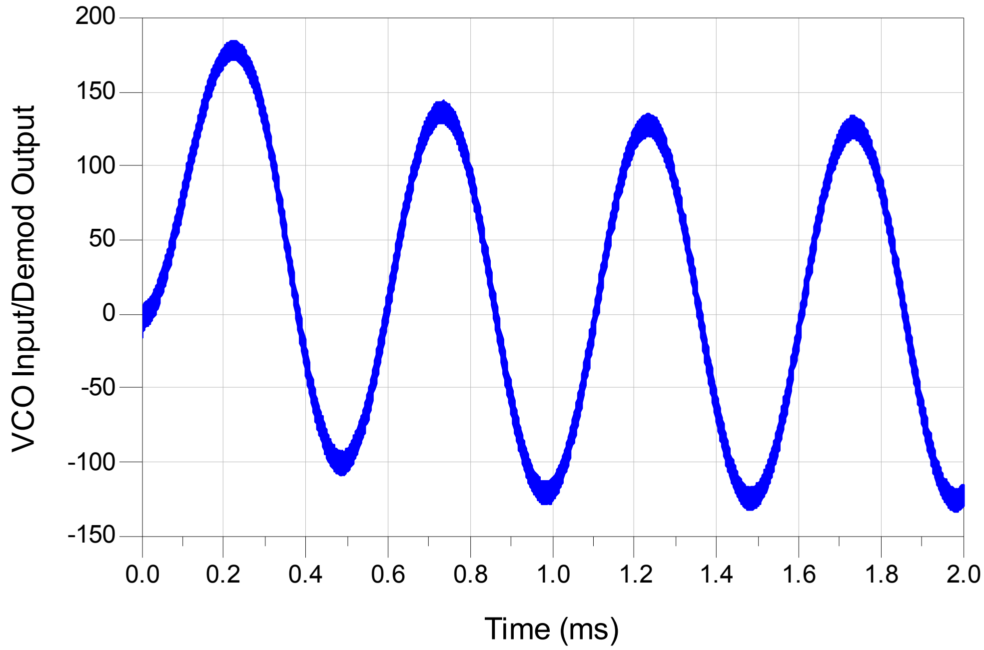

Figure 21: ADS first-order PLL demodulated FM/phase detector output/VCO input.

from 10 kHz to 20 kHz. By doubling the loop gain the lock range is doubled and the PLL should be able to track the input FM with a wider deviation. Very this experimentally.