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## Set #1

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Due Tuesday September 14, 2004

**Make note of the following:**

- Papers are due at the start of class
- Write only on one side of the paper
- Please try if possible to start each new problem on a clean sheet of paper
- Use engineering paper if you like

**Problems:**

1. Text problem 2–5.
2. Text problem 2–8.
3. Text problem 2–9.
4. Prove that if  $P(F) \geq 1 - \epsilon$  and  $P(G) \geq 1 - \epsilon$ , then also  $P(F \cap G) \geq 1 - 2\epsilon$ . In other words you are to show that if two events have probability nearly one, then their intersection has probability nearly equal to one as well.
5. Text problem 2–19.
6. Two balls are drawn from an urn containing  $n$  balls numbered from 1 to  $n$ . The first ball is kept if it is numbered 1, and returned otherwise. Show that the probability of the ball being numbered 2 is given by

$$P(D2 = 2) = \frac{n^2 - n + 1}{n^2(n - 1)}$$

7. An internet user  $A$  would like to send an important message to user  $B$ . Assume that there are two links from  $A$  to  $B$ , each with 6 relay nodes connected in series. A message may reach  $B$  through a link only if every relay node in that link is operational. Assume that a node is operational with probability 0.99, which is independent of any other node. Answers are given after each question, but you must show how to get them.
  - a.) Find the probability that one link is operational (regardless of the other link). Answer: 0.9415.
  - b.) Find the probability that at least one link is operational. Answer: 0.9966.
  - c.) Find the probability that both links are operational. Answer: 0.8864.
  - d.) What is the probability that a message will reach  $B$  if it is sent through a link chosen at random? Answer: 0.9415.
  - e.) What is the probability that a message will reach  $B$  if it is sent through both links? Answer: 0.9966.

8. Text problem 2–20.
9. Text problem 2–27.
10. **For 20 bonus points:** In a particular computer communications network, the host computer broadcasts a packet of data (say  $L$  bytes long) to  $N$  receivers<sup>1</sup>. The host computer then waits to receive an acknowledgment (ACK) message from each of the  $N$  receivers before broadcasting the next packet. If the host does not receive all of the ACKs within a certain time period, it will rebroadcast (retransmit) the same packet. The host computer is then said to be in the “retransmission mode”. It will continue retransmitting until all  $N$  ACKs are received. Then it will proceed to broadcast the next packet.

Let  $p \equiv P[\text{successful transmission of a single packet to a single receiver along with successful acknowledgement}]$ . Assume that these events are independent for different receivers or separate transmission attempts. Due to random impairments in the transmission media and the variable condition of the receivers (terminals or PCs), we have that  $p < 1$ .

- a.) In a fixed protocol or method of operation, we require that all  $N$  of the ACKs be received in response to a given transmission attempt for that packet transmission to be declared successful. Let the event  $S(m)$  be defined as follows:  $S(m) \equiv \{\text{a successful transmission of one packet to all } N \text{ receivers in } m \text{ or fewer attempts}\}$ . Find the probability

$$P(m) \equiv P[S(m)].$$

(Hint: Consider the complement of the event  $S(m)$ .)

- b.) An improved system operates according to a dynamic protocol as follows. Here we relax the ACK requirement on retransmission attempts, so as to only require acknowledgments from those receivers that have not yet been heard from on previous attempts to transmit the current packet. Let  $S_D(m)$  be the same event as in part (a) but using the dynamic protocol. Find the probability

$$P_D(m) \equiv P[S_D(m)].$$

(Hint: First consider the probability of the event  $S_D(m)$  for an individual receiver, and then generalize to the  $N$  receivers.)

Note: If you try  $p = 0.9$  and  $N = 5$  you should find that  $P(2) < P_D(2)$ , that is  $P(3) = 0.93133$  and  $P_D(3) = 0.99501$ .