ECE 6650 Midterm October 6, 2014 Name:

Take-Home **Exam Honor Code**

This being a take-home exam a strict honor code is assumed. Each person is to do his/her own work with no consultation with others regarding these problems. Bring any questions you have about the exam to me. Please be clear and concise in your answers. For MATLAB related problems include copies of any functions you have written. The exam is due on Monday October 20, 2014.

Analysis Only Problem:

1.) Text problem 4.8.

MATLAB/Computer Related Problems:

2.) As the first step in this exercise you will develop several algorithms for time series analysis. The MATLAB signal processing toolbox does supply these function, but the desire here is to to write your own, to better understand how they work. Assume that you are given the time series $u(n)$ for $n = 1, 2, \ldots, N$. This fits well with MATLAB vector notation. Code the following algorithms in MATLAB for complex data:

i. The biased autocorrelation function estimate

$$\hat{r}(k) = \frac{1}{N}\sum_{n=k+1}^{N} u(n)u^*(n-k), k = 0, 1, \ldots, M < N$$

ii. The Levinson-Durbin recursion for finding

$$\kappa_m, P_m, \text{ and } a_{m,k}, k = 0, 1, \ldots, M$$

for $m = 1, 2, \ldots, M$.

iii. The Burg algorithm for finding

$$\kappa_m, P_m, \text{ and } a_{m,k}, k = 0, 1, \ldots, M$$

for $m = 1, 2, \ldots, M$.

Test your routines using the following time series:

a.) An AR(2) process defined as

$$u(n) - 0.1u(n-1) - 0.8u(n-2) = v(n)$$

where $\sigma_v^2 = 0.27$. Choose $M = 2$ (why?) and run experiments to determine the AR parameters with $N = 10, 100,$ and, 500 using both algorithms. Be sure to eliminate transients in the AR process (filter output) by running the filter for a while before collecting samples for the estimation algorithms. Summarize your results by placing them in a table for easy comparison. Comment on algorithm performance.

b.) An $N = 500$ point record of an *unknown* real AR process is available as the ANSII text file ar_proc.dat. This file is located in a ZIP file on the web site along with this exam document. Determine experimentally, using both algorithms, M, $\mathbf{a}_m$, and $\sigma_v^2 = P_M$. Assume that $M < 10$. Plot the estimated AR power spectrum in each case, i.e.,

$$P_{\text{AR}}(\omega) = \frac{\sigma_v^2}{\left|\sum_{k=0}^{M} a_{M,k}e^{-j\omega k}\right|^2}$$

You may of course use `freqz()` to compute the spectrum function. To determine the approximate AR process order from measurements, use either the *Akaike information cri-*

terion (AIC), the *final prediction error* (FPE), or the *minimum description length* (MDL). The idea is to choose the value for M that minimizes these quantities. For short data records the AIC is given by

$$\mathrm{AIC}(m) = N\ln(\hat{P}_m) + 2m$$

where $\hat{P}_M$ is the estimated prediction error power at order m. The FPE is given by

$$\mathrm{FPE}(m) = \frac{N+m}{N-m}\hat{P}_m$$

The MDL is given by

$$\mathrm{MDL}(m) = N\log\hat{P}_m + m\log N$$

When $m \ll N$ (typically the case) a clear minimum is not indicated by the above functions, so determination of the exact model order is still not exact.

3.) Text problem 4.10. Here you determine bounds on the step size parameter μ. This problem will now be expanded to explore steepest descent performance similar to text Section 4.4/ notes Chapter 4 Example 1. To make the algorithm function in a more reasonable fashion assume now that the AR(2) coefficients are $a_1 = 0.5$ and $a_2 = 0.95$. For two values of μ less than this maximum plot the weight vector trajectory for an $M = 2$ estimation of this process similar to that done in Section 4.4 of the text, and also in Example 1 of the Chapter 4 notes. The MATLAB function given in the notes will be very useful here. Also plot the learning curves for each of the two μ values you choose. As in the book/notes example choose as the tap weight vector starting point

$$\mathbf{w}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$